

*5th Meeting of the Spanish Section of the
Institute of Combustion*



**BURNING VELOCITY OF CONCAVE
(NEGATIVELY STRETCHED) FLAME TIPS:
EXPERIMENTS AND MODELING**

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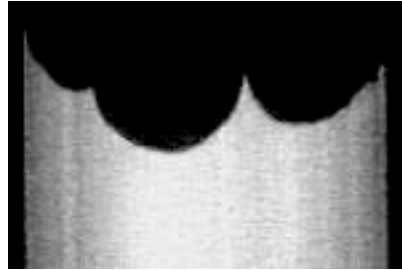
Santiago de Compostela, May 23 – 25, 2011

CONTENT

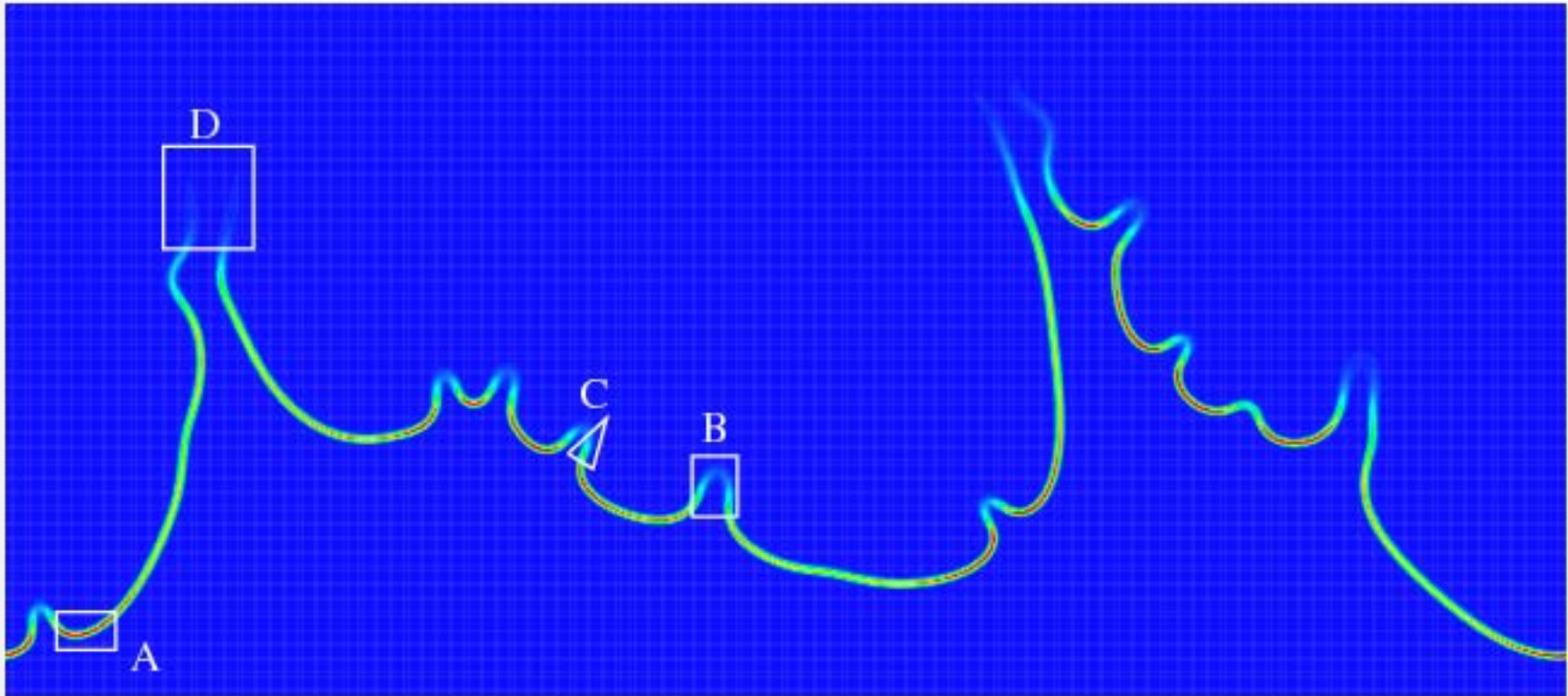
- Introduction and Motivation.
- Numerical techniques
- Experimental facility
- .Gas velocity field.
- Flame tomography and mean curvature
- Flame speed and stretch
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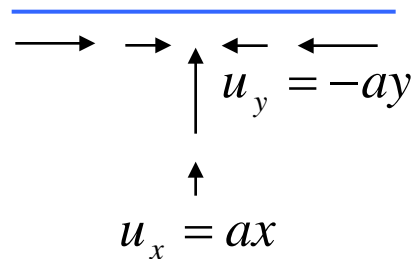
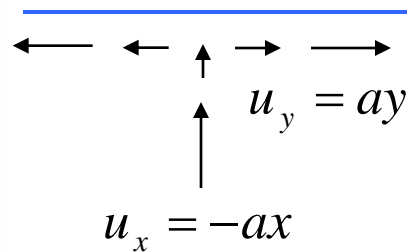
Cellular flame
(IRPHE, Marseille)



(CCSE – Berkeley)

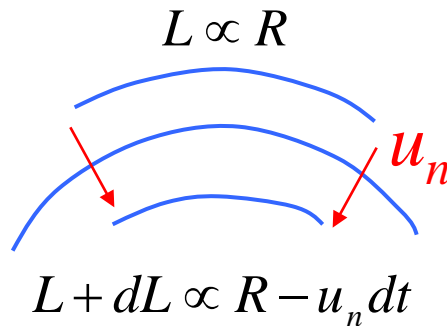
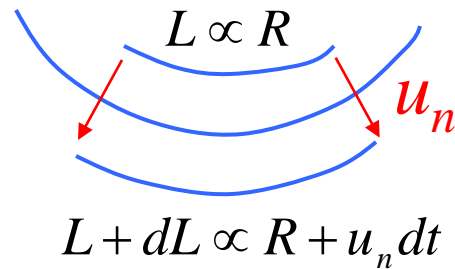
Strained flame

$$\frac{1}{L} \frac{dL}{dt} = \frac{\partial u_s}{\partial s}$$



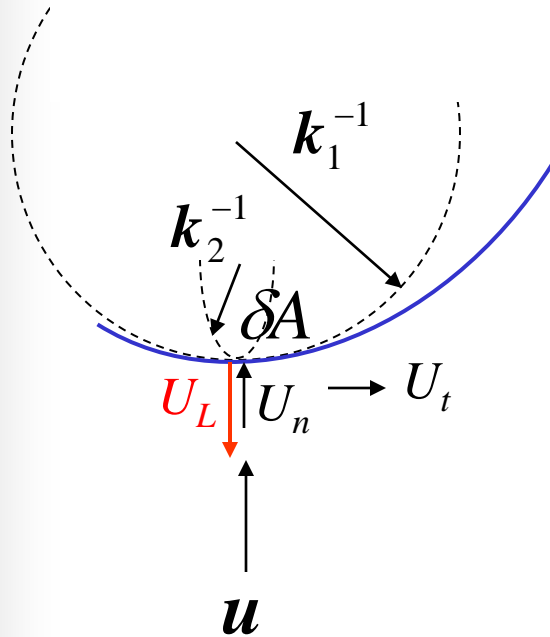
Curved flame

$$\frac{1}{L} \frac{dL}{dt} = \pm \frac{u_n}{R}$$



Positive stretch
(Front extension)

Negative stretch
(Front compression)



$\mathcal{L} = \text{Markstein length}$

$$\frac{U_n}{U_L} - 1 = - \frac{\mathcal{L}}{U_L} \frac{\delta \dot{A}}{\delta A}$$

$S = \text{Total stretch}$

$$\frac{1}{\delta A} (\delta \dot{A}) = -U_L \nabla \cdot \mathbf{n} - \mathbf{n} \cdot \nabla \mathbf{u} \cdot \mathbf{n} \quad (\text{Clavin \& Joulin, 1983})$$

Front curvature

$$\nabla \cdot \mathbf{n} = k_1 + k_2$$

Flow straining

$$Ma \equiv \mathcal{L} / \delta$$

$$\text{Markstein number} \equiv \frac{\text{Markstein length}}{\text{flame thermal thickness } (\lambda / \rho c_p U_L)}$$

Activation Energy Asymptotic Methods: $Ma = \frac{1}{\gamma} J + \beta (Le - 1) \frac{1 - \gamma}{2\gamma} D$
 (Clavin & G-Y, 1983)

$$J = \frac{\gamma}{1 - \gamma} \int_0^1 h(\theta) \frac{1}{1 + \theta\gamma / (1 - \gamma)} d\theta$$

$$D = -\frac{\gamma}{1 - \gamma} \int_0^1 h(\theta) \frac{\ln \theta}{1 + \theta\gamma / (1 - \gamma)} d\theta$$

$$\theta \equiv (T - T_u) / (T_b - T_u)$$

$$\beta \equiv E_a / RT_u$$

$$\gamma \equiv (T_b - T_u) / T_b$$

$$h(\theta) \equiv \frac{\lambda / c_p}{(\lambda / c_p)_u}$$

GENERALIZED RELATION

(Clavin & Joulin, 1988)

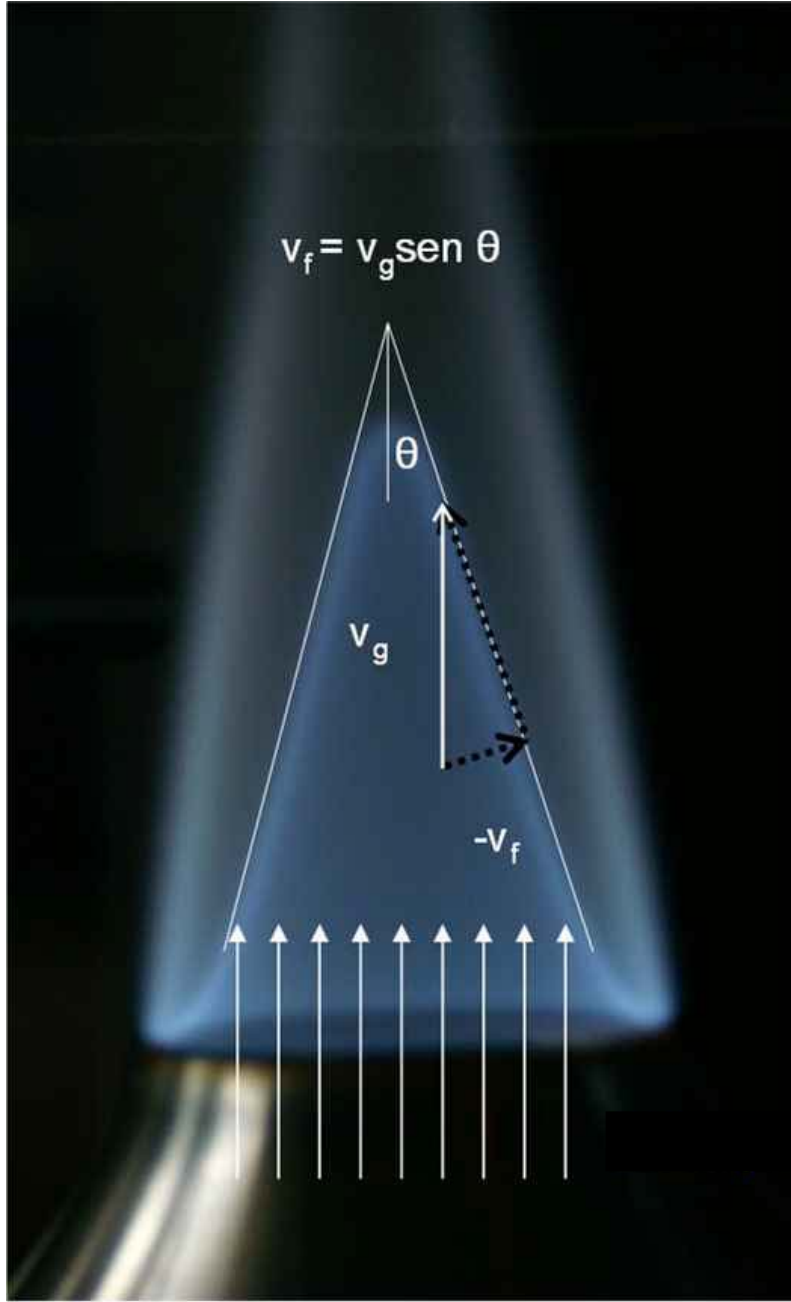


Different effects have different Markstein lengths

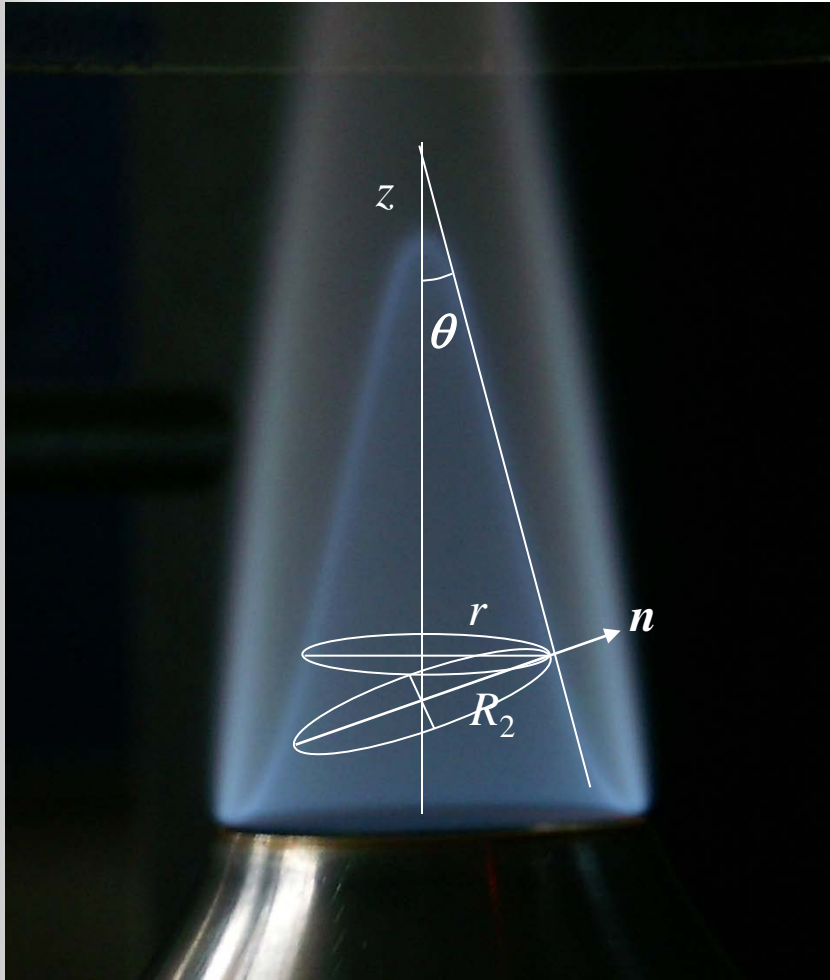
$$\frac{U_n}{U_L} - 1 = \mathcal{L}_c \nabla \cdot \mathbf{n} + \mathcal{L}_s \frac{1}{U_L} \mathbf{n} \cdot \nabla \mathbf{u} \cdot \mathbf{n}$$

Linear theory leads to: $\mathcal{L} = \mathcal{L}_c = \mathcal{L}_s$

LAMINAR JET BURNER (BUNSEN) FLAME



R. W. Bunsen (1811-99)



1. Curvature of the 2-D section:

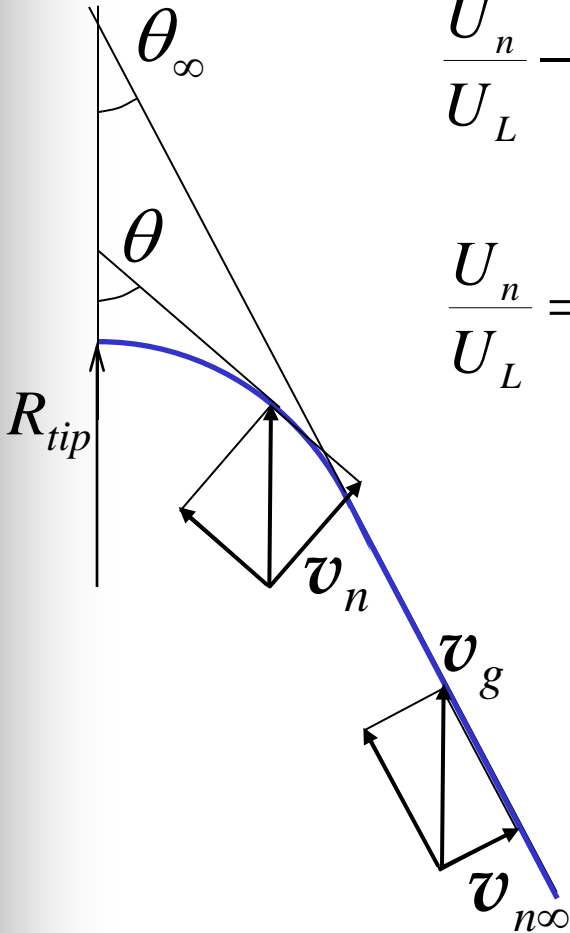
$$z_f = z(r), \quad R_1 = -\frac{(1 + z'^2)^{3/2}}{z''}$$

$$\sin \theta = \frac{1}{\sqrt{1 + z'^2}}, \quad \cos \theta = -\frac{z'}{\sqrt{1 + z'^2}}$$

2. Curvature due to axial symmetry:

$$\text{Moivre's formula: } R_2 = \frac{r}{\cos \theta}$$

CURVED (NON STRAINED) JET FLAME

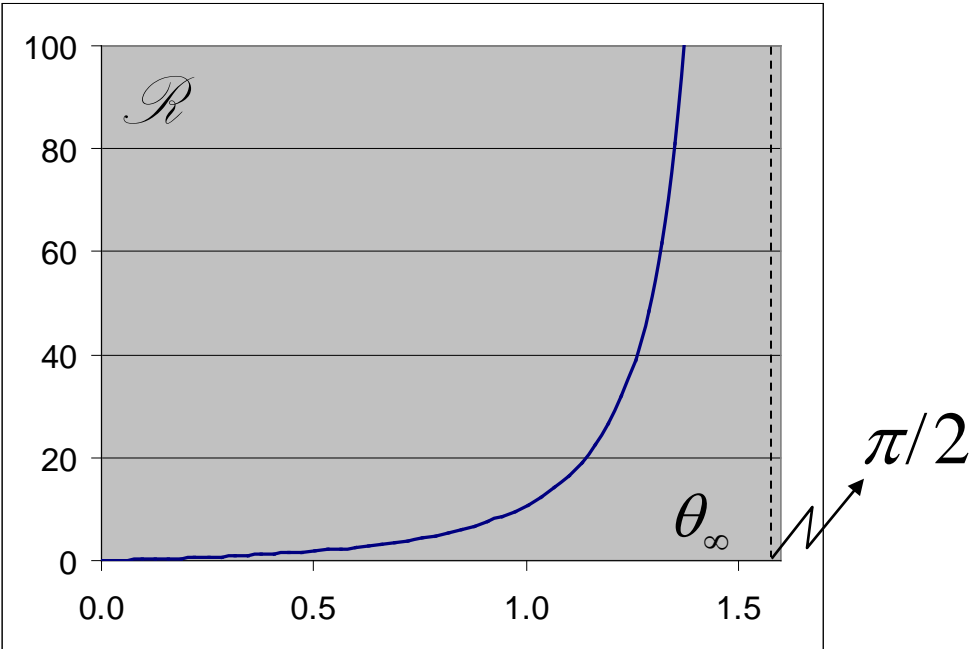


$$\left. \begin{aligned} \frac{U_n}{U_L} - 1 &= \mathcal{L} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \\ \frac{U_n}{U_L} &= \frac{v_g \sin \theta}{U_L} = \frac{\sin \theta}{\sin \theta_\infty} \end{aligned} \right\}$$

$$\frac{1}{\sin \theta_\infty} - 1 = \mathcal{L} \frac{2}{R_{tip}}$$

$$\mathcal{R} \equiv R_{tip} / \mathcal{L} = \frac{2 \sin \theta_\infty}{1 - \sin \theta_\infty}$$

$$v_{n\infty} \cong U_L \Rightarrow \frac{v_g}{U_L} = \frac{1}{\sin \theta_\infty}$$



$$\frac{U_n}{U_L} - 1 = \mathcal{L} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$z' \equiv \frac{dz}{dr} \equiv \frac{d(z/\mathcal{L})}{d(r/\mathcal{L})} \equiv \frac{d\zeta}{d\rho} \equiv \dot{\zeta}$$

$$R_1/\mathcal{L} = -\frac{(1 + \zeta^2)^{3/2}}{\dot{\zeta}}$$

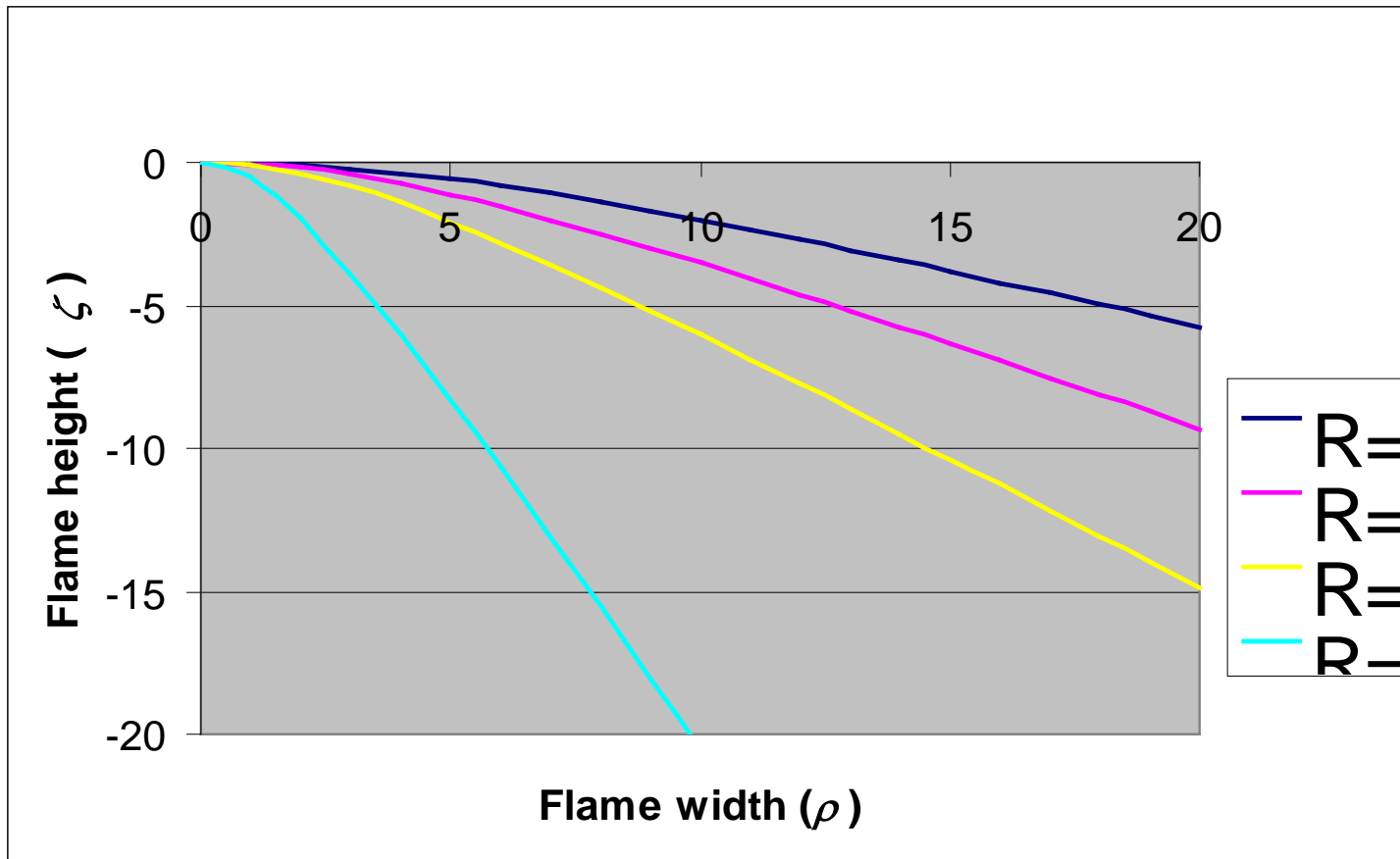
$$R_2/\mathcal{L} = -\frac{r\sqrt{1 + \zeta^2}}{\dot{\zeta}}$$

$$\ddot{\zeta} = \left(\sqrt{1 + \zeta^2} - \frac{1}{\sin \theta_\infty} - \frac{\dot{\zeta}}{\rho} \right) (1 + \zeta^2)$$

$$\zeta(0) = 0$$

$$\dot{\zeta}(0) = 0$$

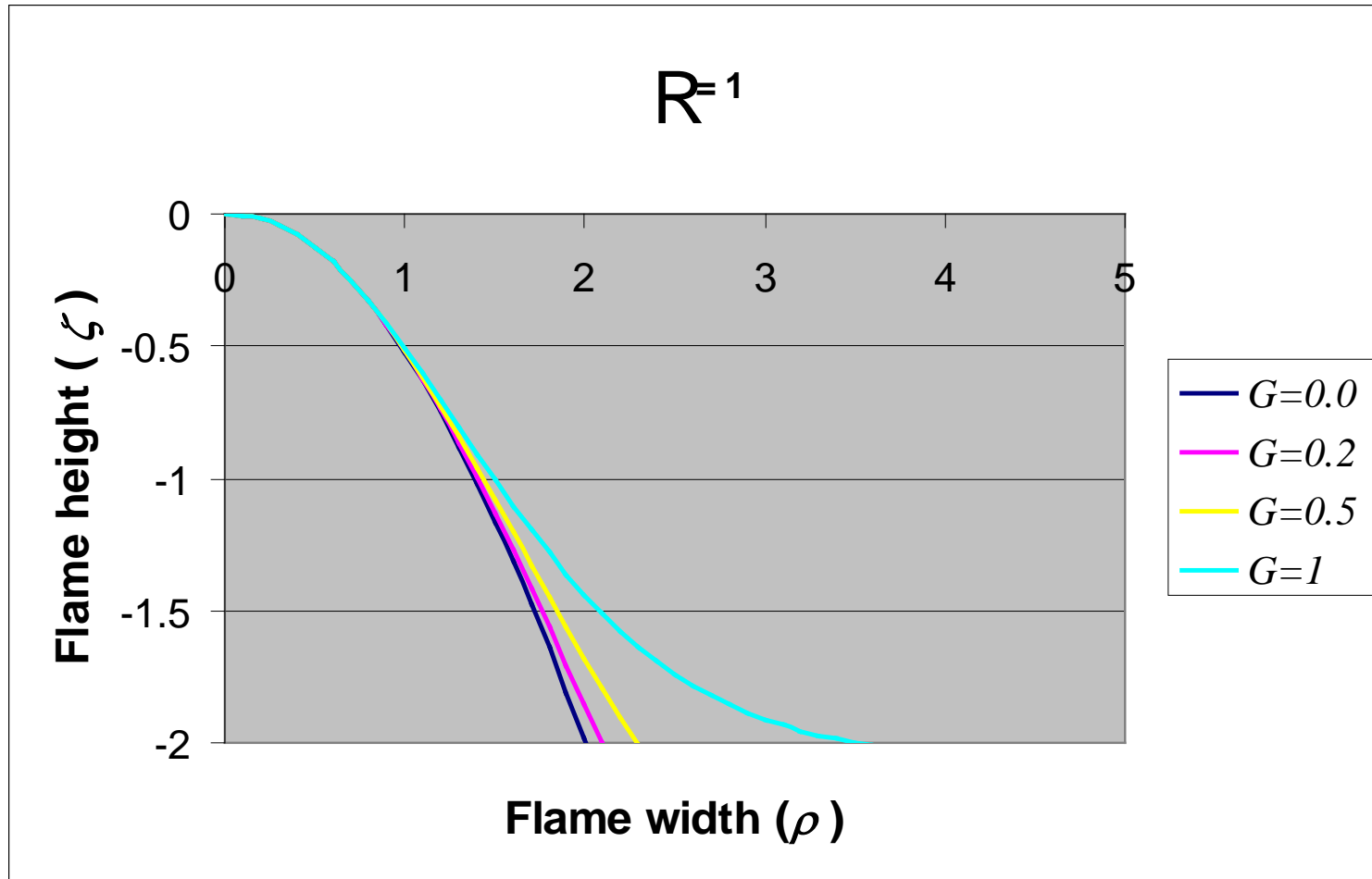
SHAPE OF A CURVED JET FLAME

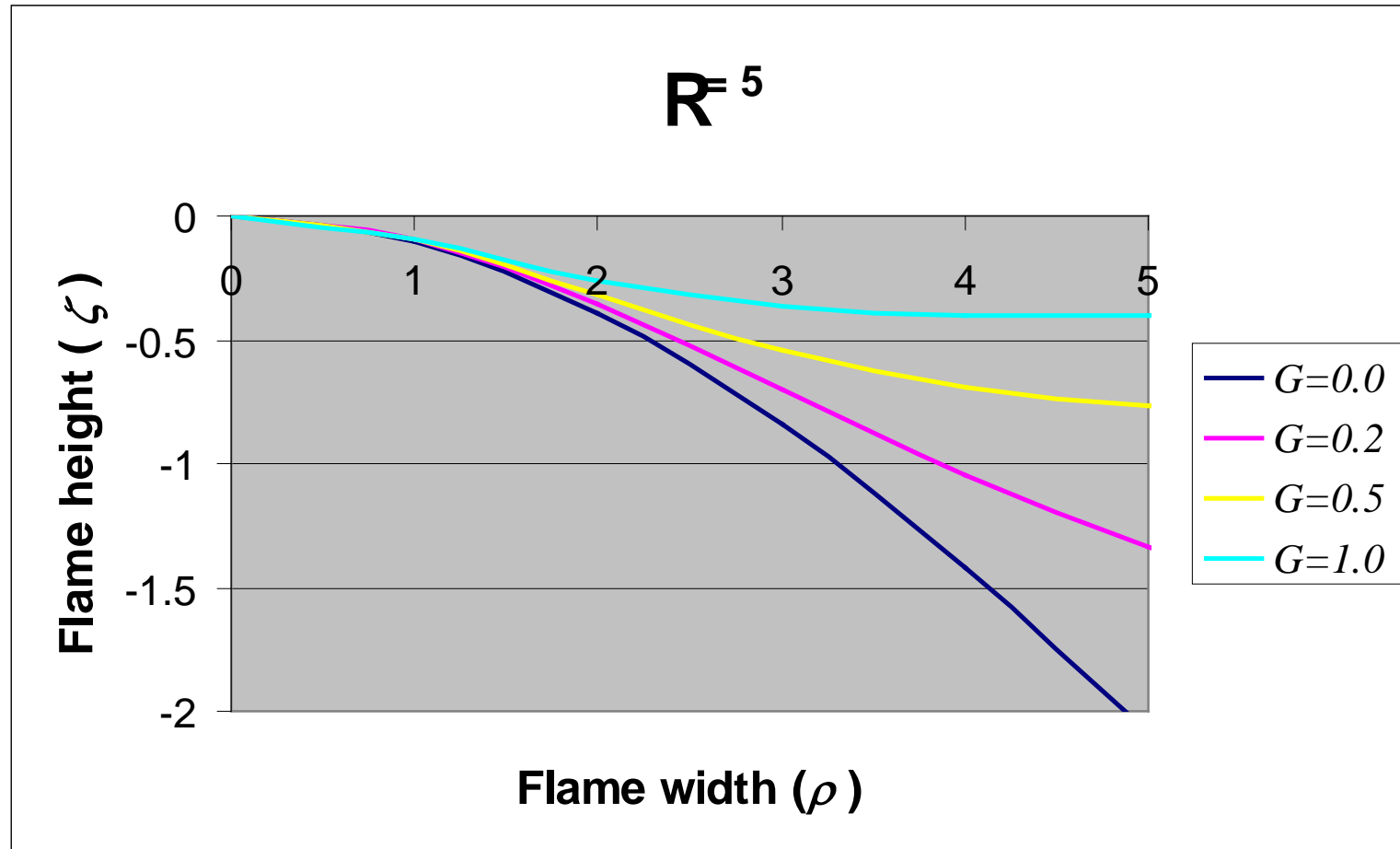


$$\frac{\mathbf{v}_g}{U_L} = \left(-\frac{G}{2}\rho, 0, \frac{v_{tip}}{U_L} + G\zeta \right) \Rightarrow \mathbf{n} \cdot (\nabla \mathbf{v}_g) \cdot \mathbf{n} = G \frac{1 - \dot{\zeta}^2 / 2}{1 + \dot{\zeta}^2}$$

$$\frac{U_n}{U_L} = - \left(\frac{\mathbf{v}_g}{U_L} \cdot \mathbf{n} \right) = \frac{G(\zeta + \rho \dot{\zeta} / 2) + v_{tip} / U_L}{\sqrt{1 + \dot{\zeta}^2}}$$

$$\left\{ \begin{array}{l} \ddot{\zeta} = \left[\sqrt{1 + \dot{\zeta}^2} - \frac{1}{\sin \theta_\infty} - \frac{\dot{\zeta}}{\rho} + G \left(\frac{1 - \dot{\zeta}^2 / 2}{\sqrt{1 + \dot{\zeta}^2}} - \frac{\rho \dot{\zeta}}{2} - \zeta - 1 \right) \right] (1 + \dot{\zeta}^2) \\ \zeta(0) = 0 \\ \dot{\zeta}(0) = 0 \end{array} \right.$$





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Numerical techniques

- Assumptions:

- Ideal inviscid gas

- Axisymmetric

- Irreversible Arrhenius reaction

- High activation energy

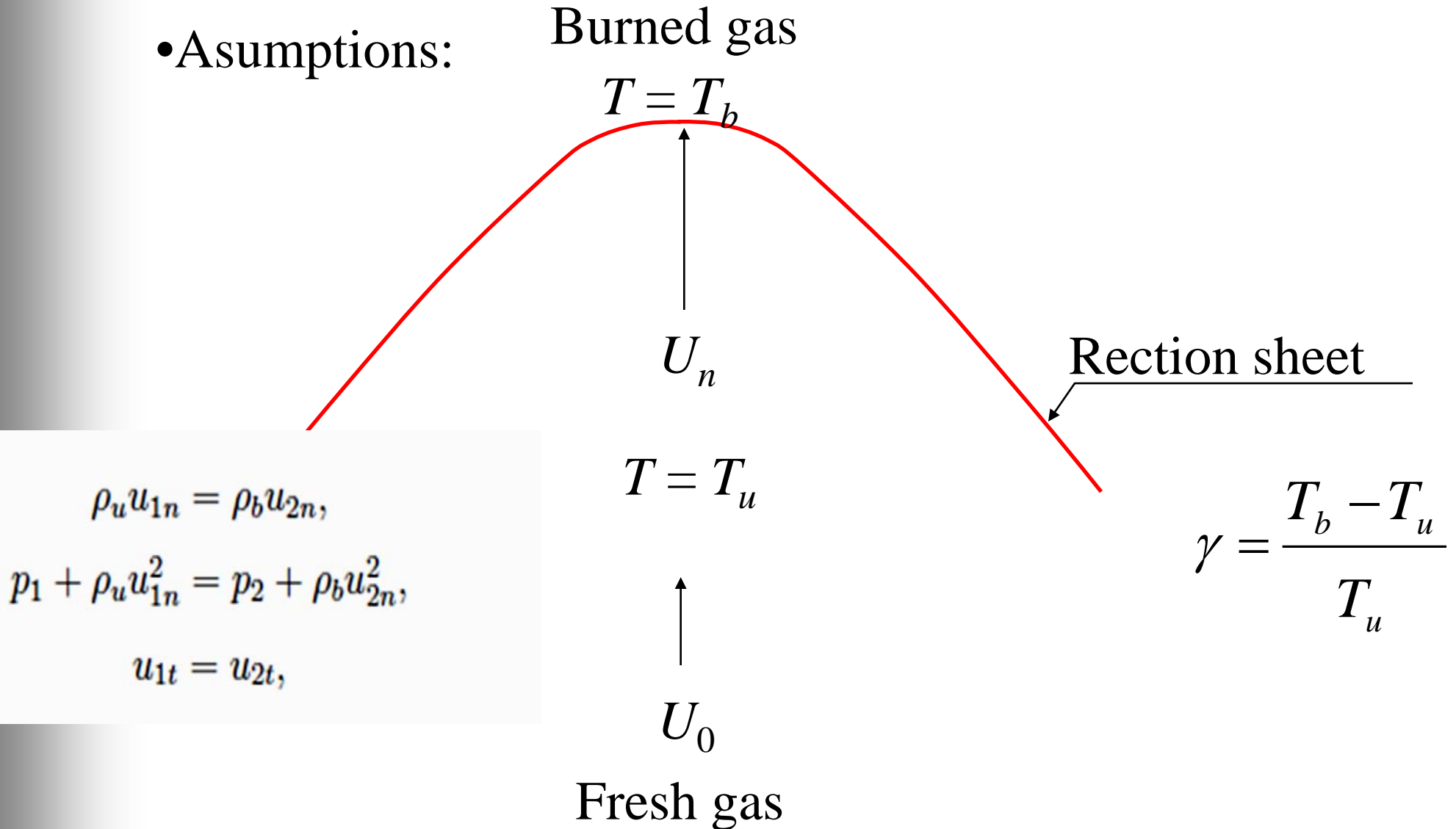
- $l_e = O(1) \Rightarrow$ reaction region non affected by curvature

- Quasi-isobaric low Mach number approximation

- No gravity

} reaction layer :
infinitely thin free
boundary

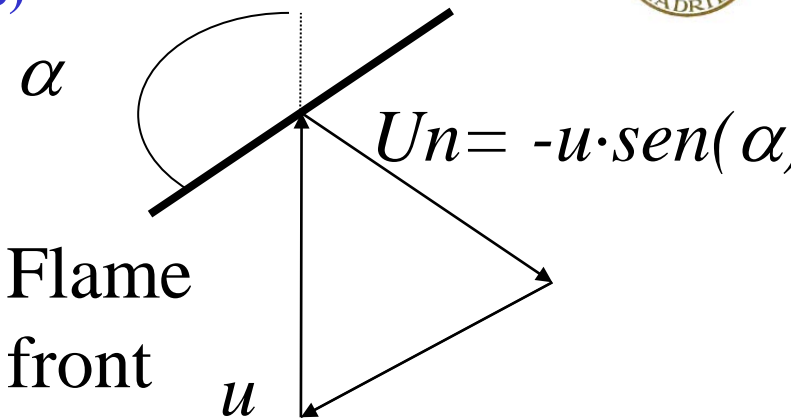
- Assumptions:



GENERALIZED RELATION

(Clavin & Joulin, 1988)

$$\frac{U_n}{U_L} - 1$$



$$\frac{U_n}{U_L} - 1 = \mathcal{L}_K (\nabla \cdot \mathbf{n}) + \mathcal{L}_G \left(\frac{1}{U} \mathbf{n} \cdot \nabla \mathbf{u} \cdot \mathbf{n} \right)$$

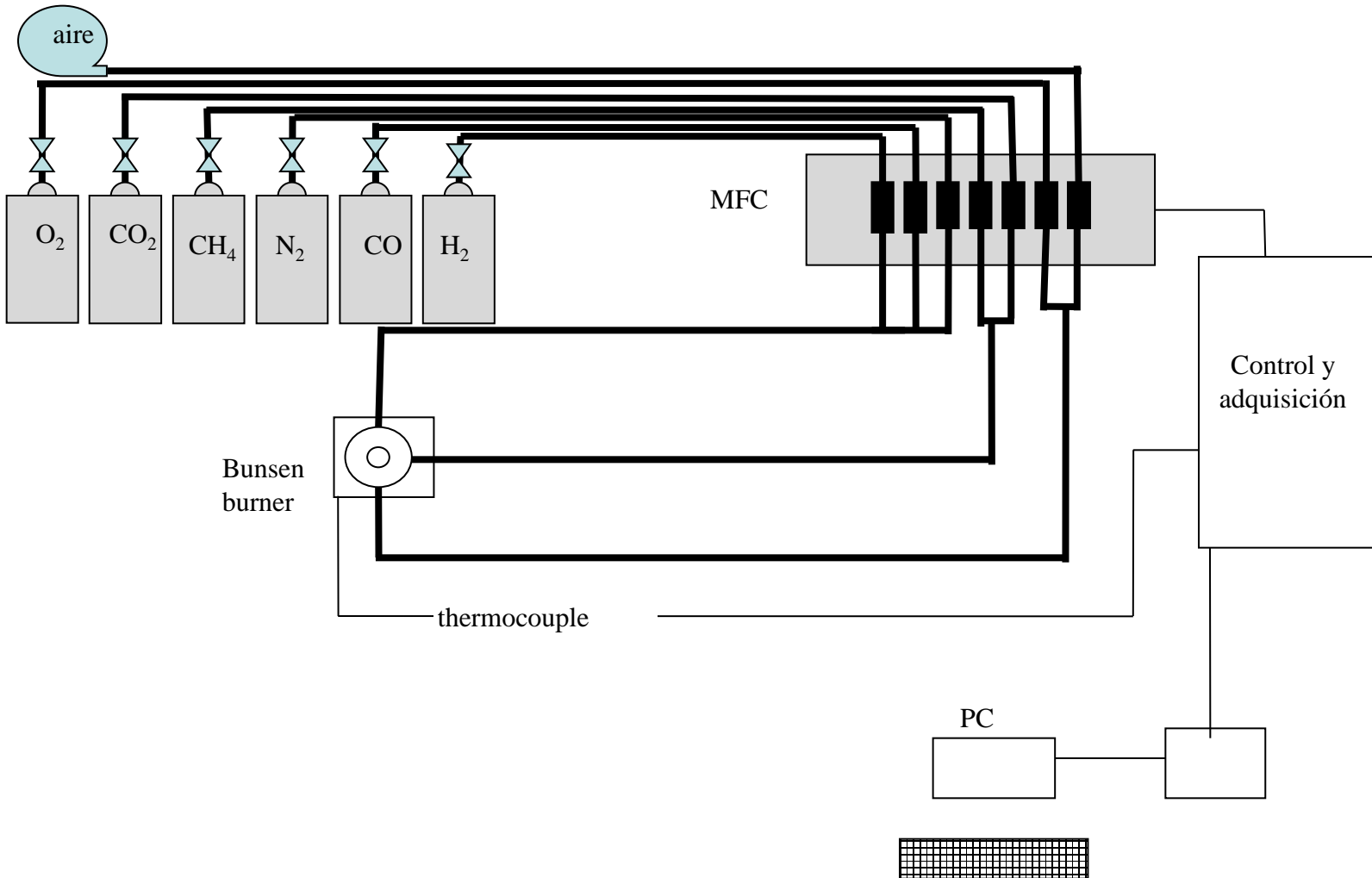
$$(\nabla \cdot \mathbf{n}) + \left(\frac{1}{U_L} \mathbf{n} \cdot \nabla \quad \mathbf{n} \right)$$

u

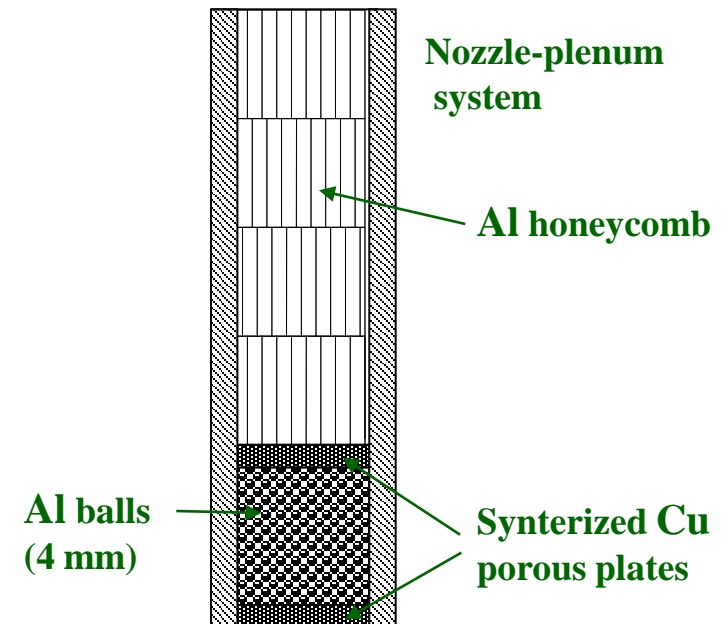
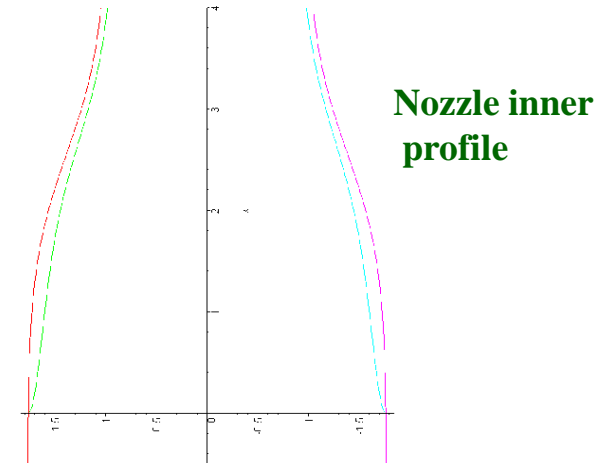
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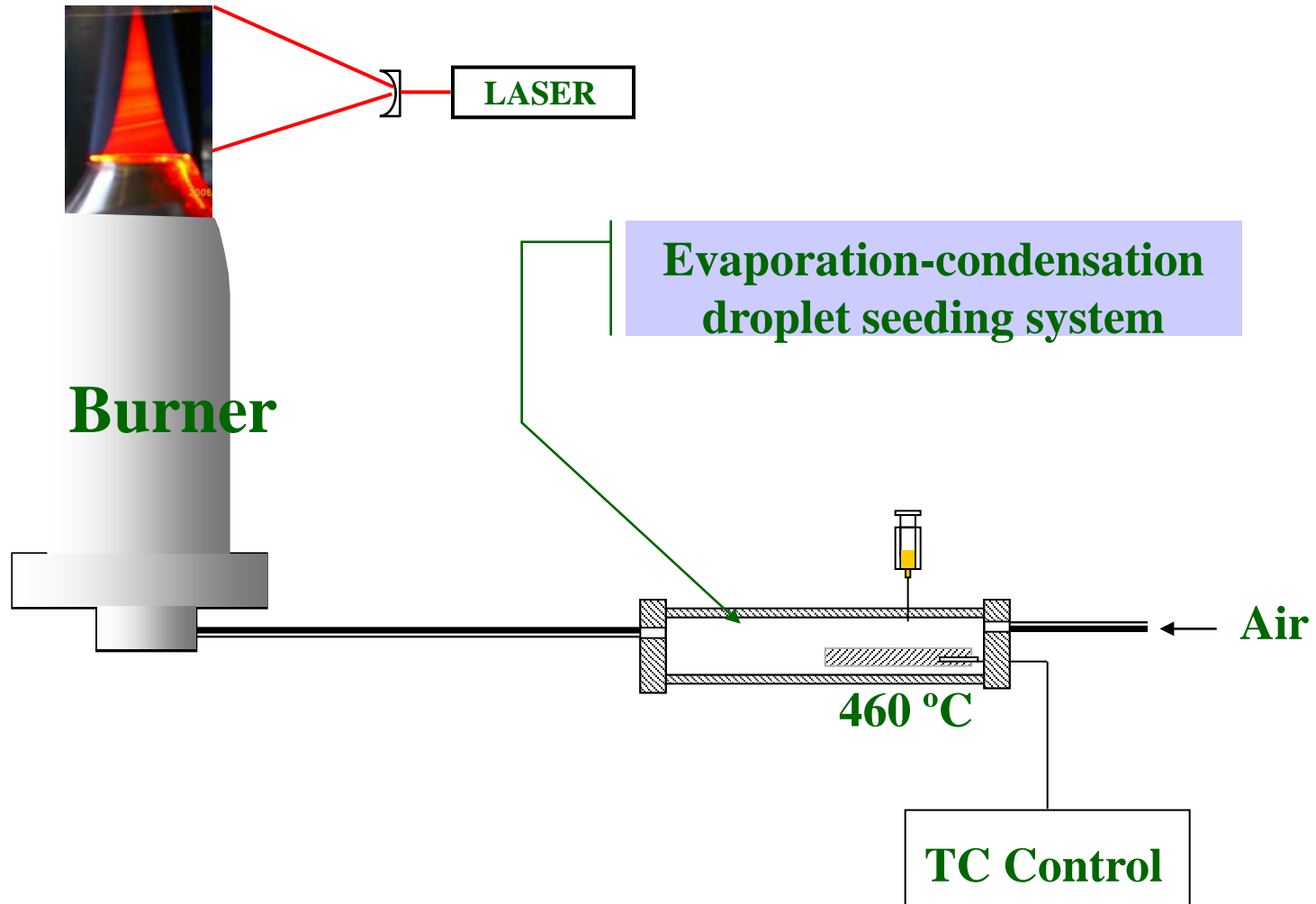
Experimental facility



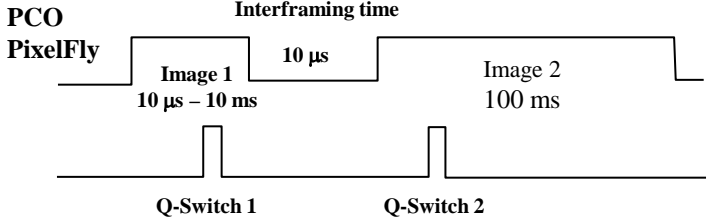
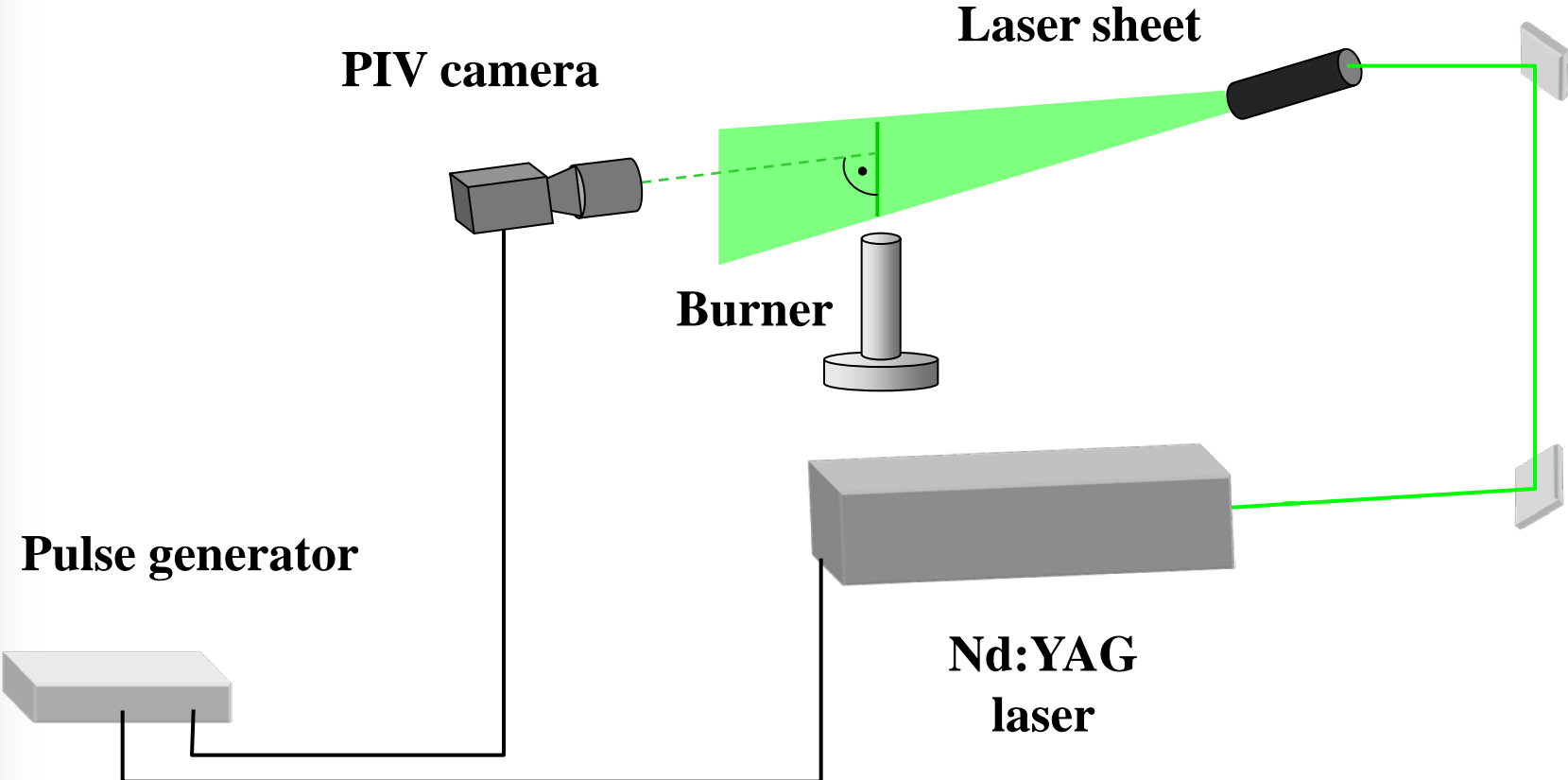
LAMINAR JET BURNER



OIL DROPLET VISUALIZATION



PIV DIAGNOSTICS SYSTEM



PARTICLES

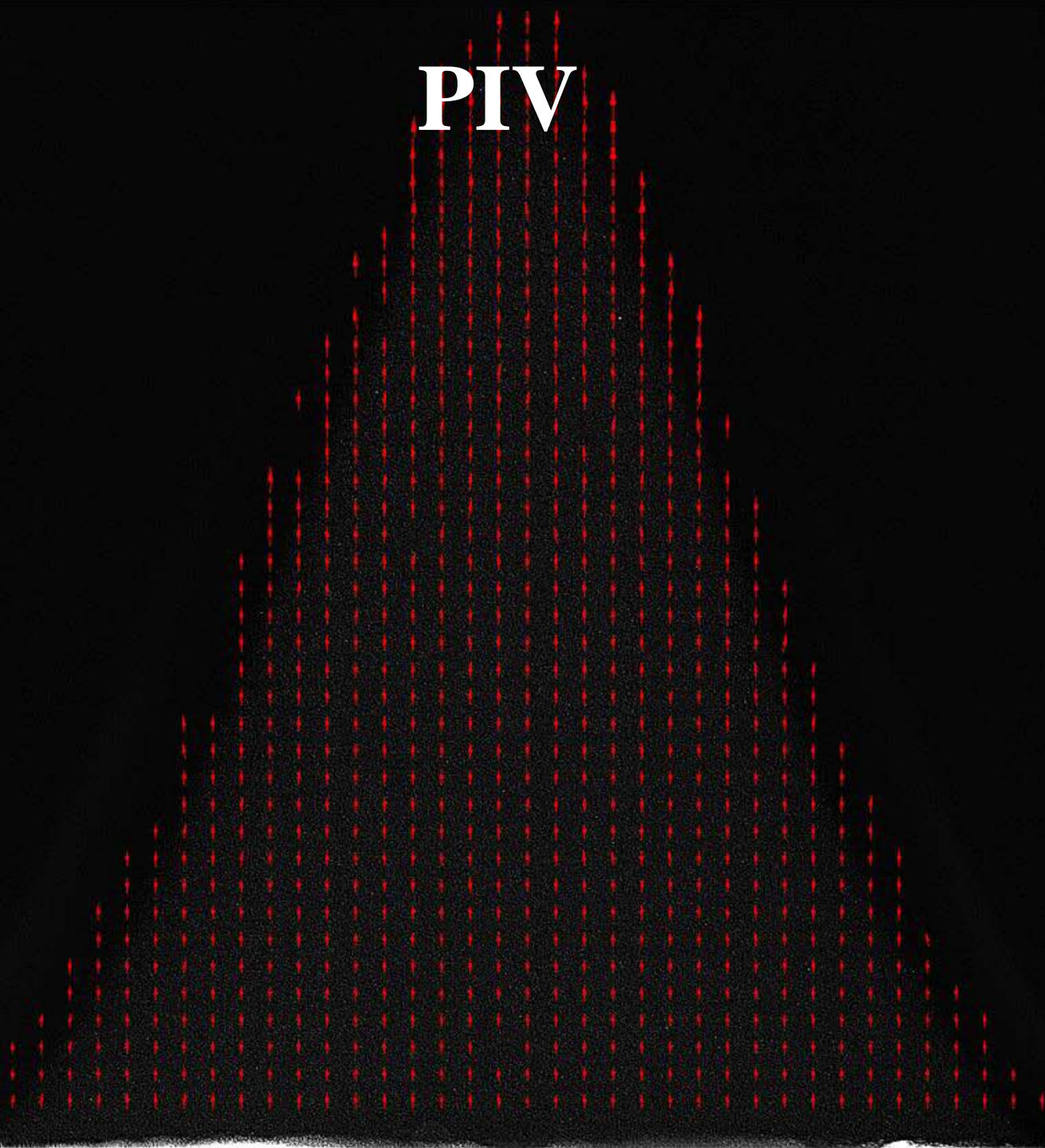
PARTICLES

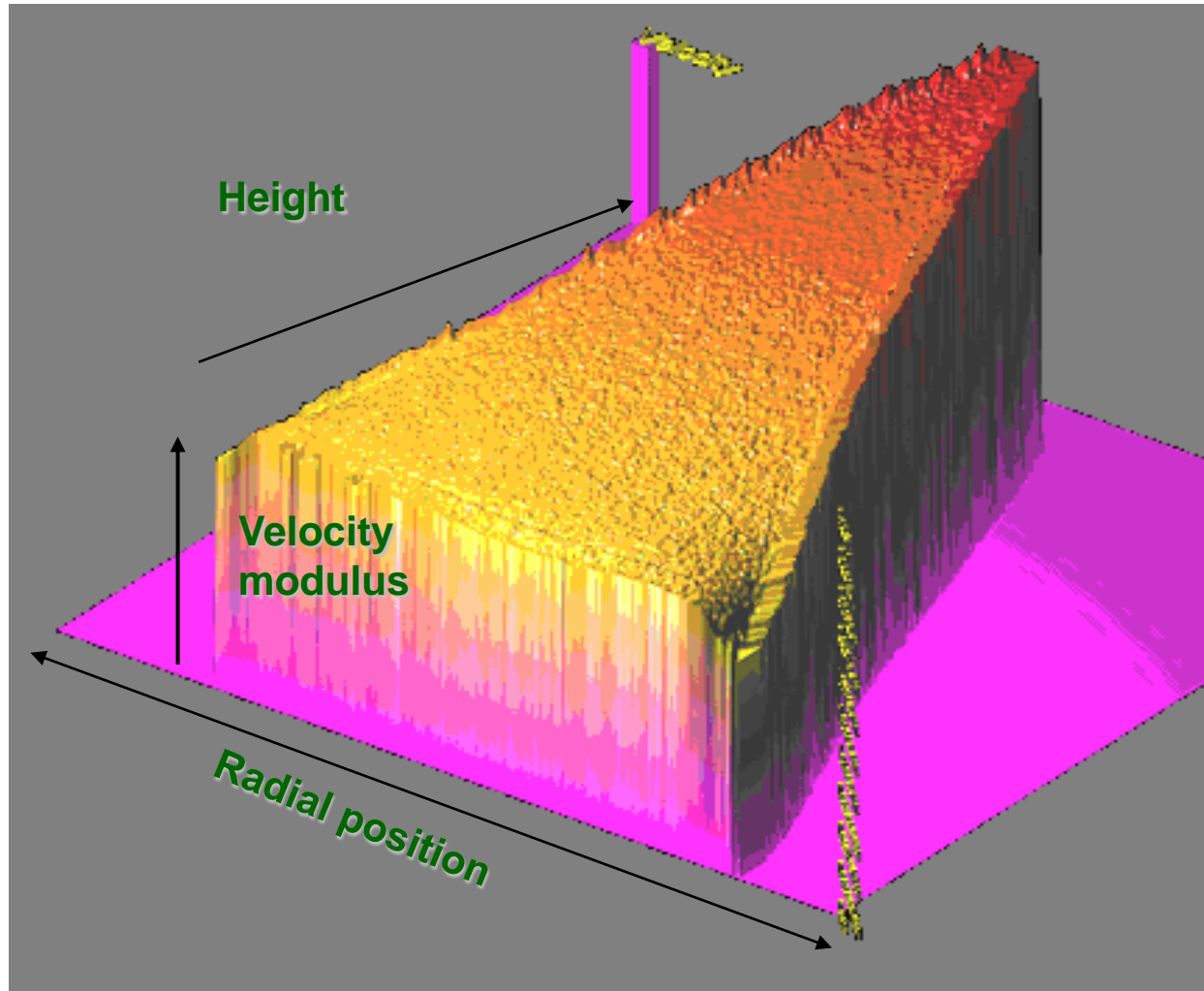


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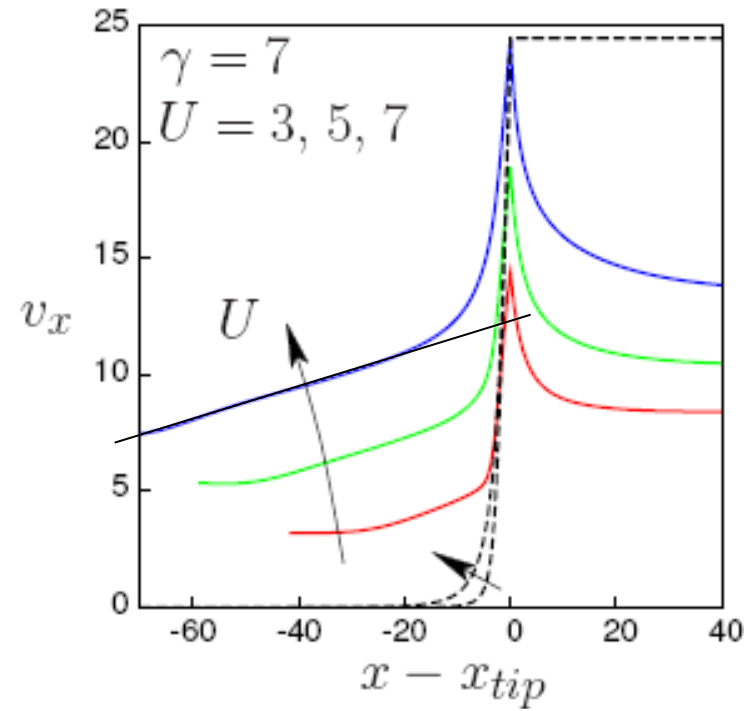
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PIV





VELOCITY ALONG THE SYMMETRY AXIS

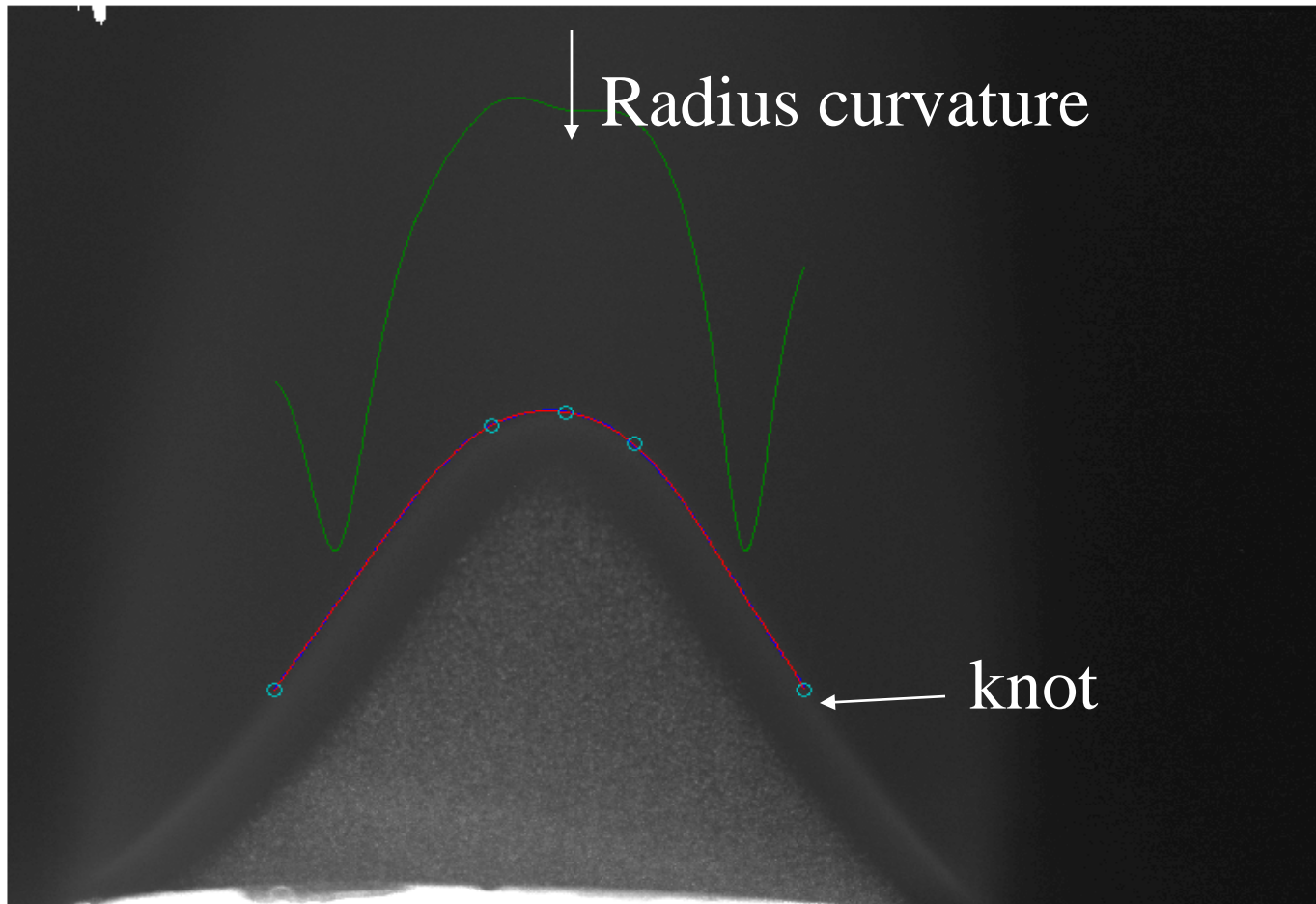


$$U \equiv U_n / U_L$$

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Minimal spline

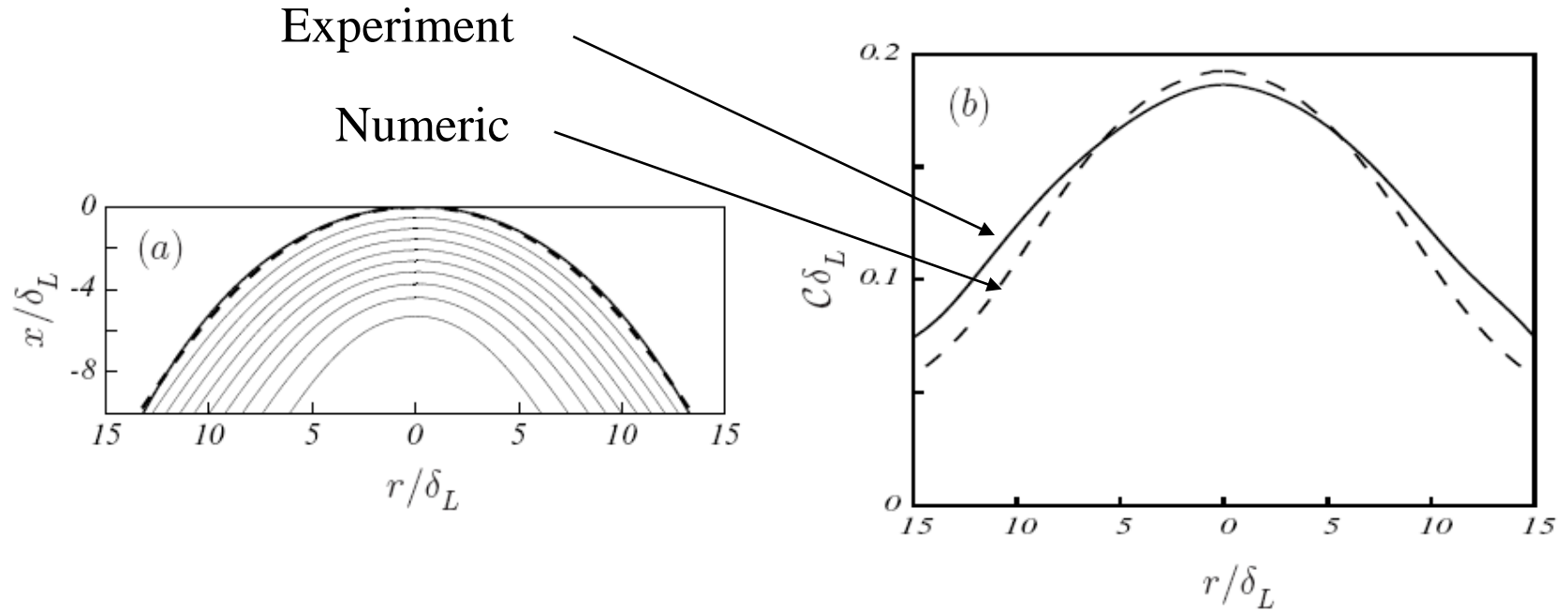


blue: detected flame front

red: detected flame front

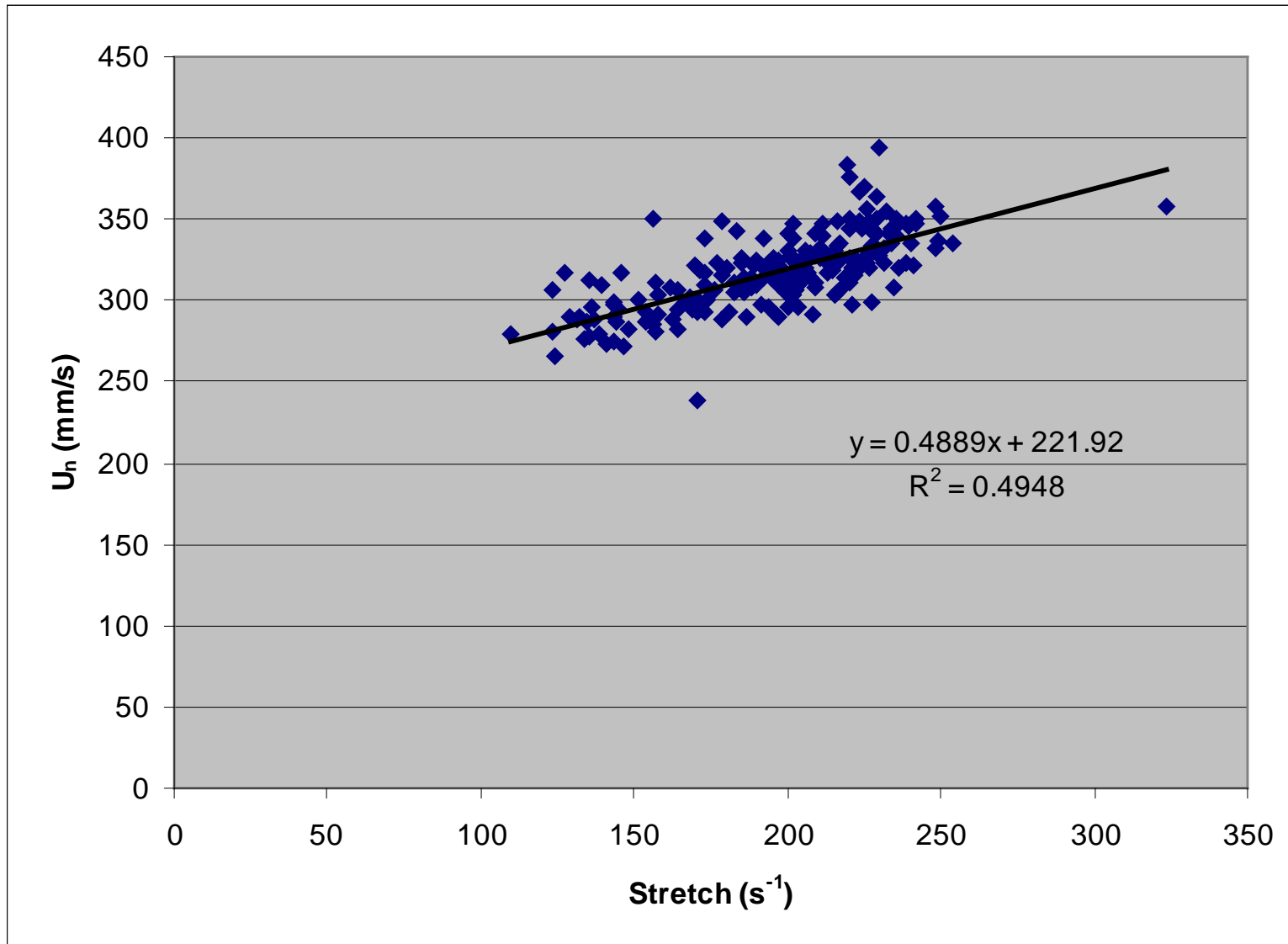
SHAPE OF THE REACTIVE SHEET

$$(\gamma = 6, U_n/U_L = 4.15)$$

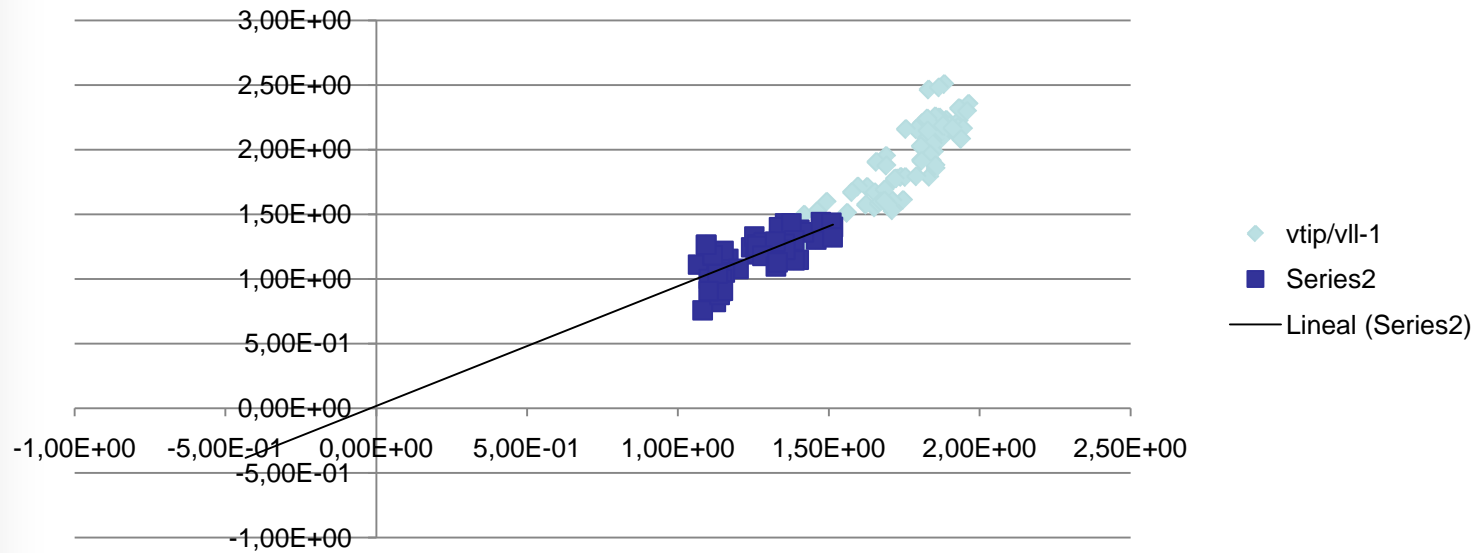


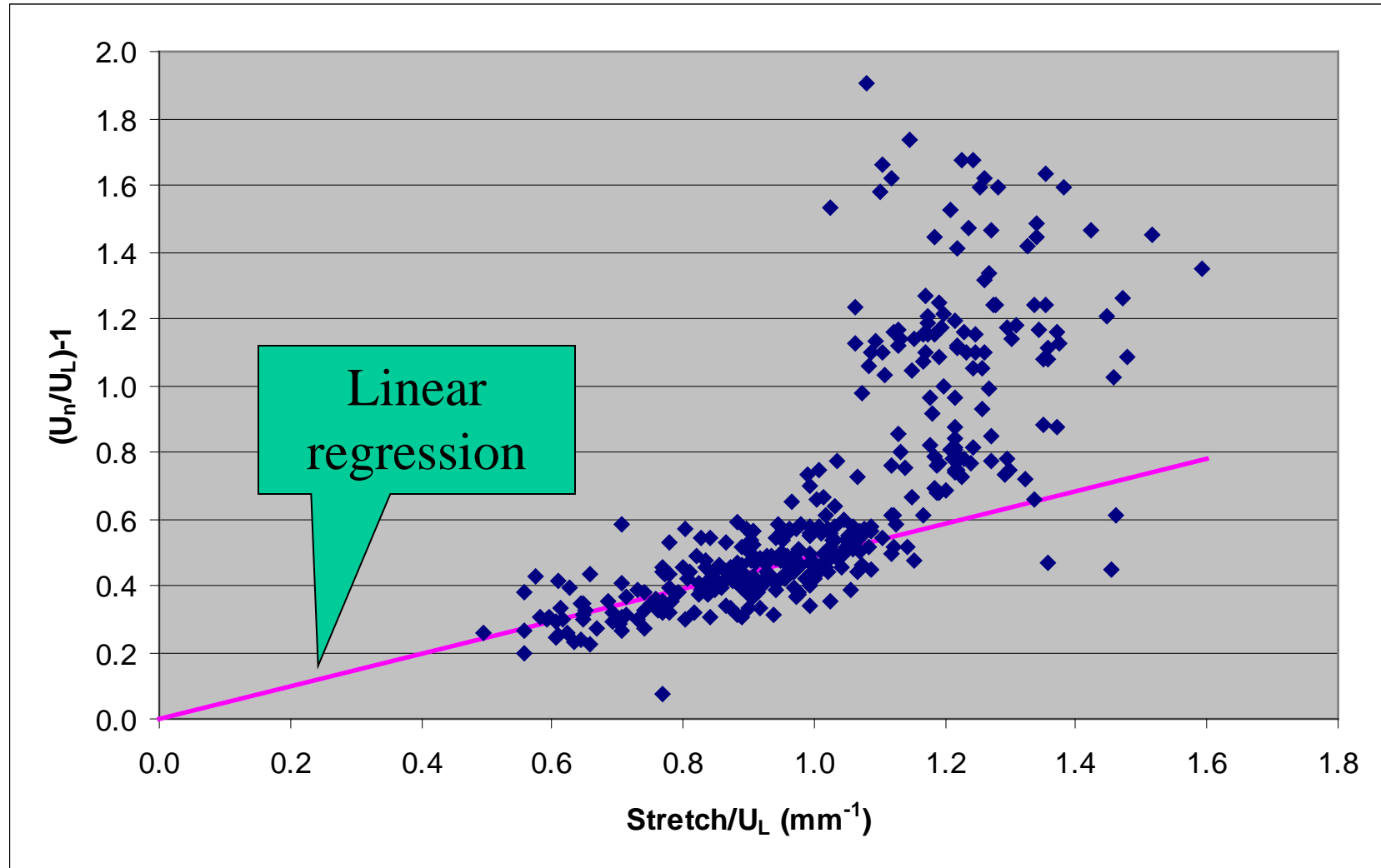
CONTENT

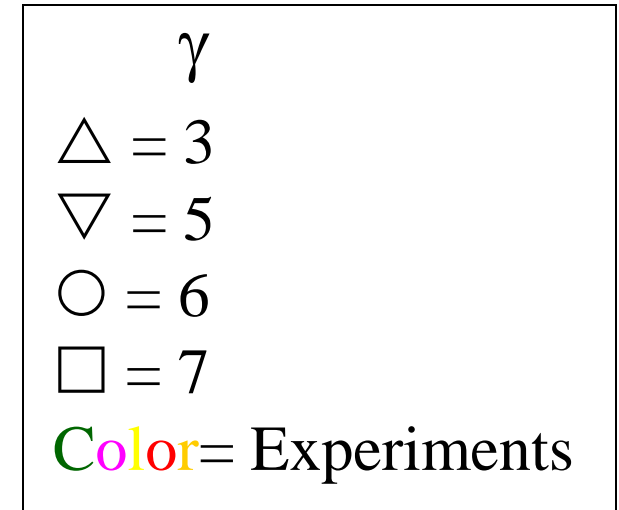
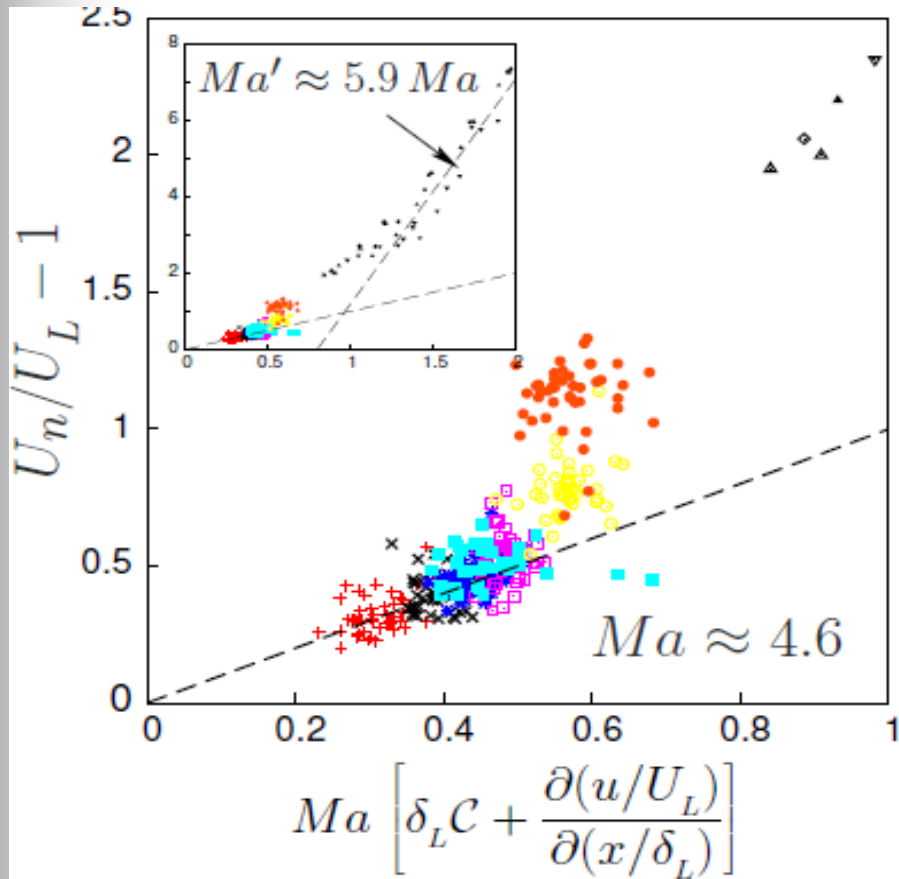
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U_n vs. S – Linear correlation (flame tip, $\phi = 1.43$)

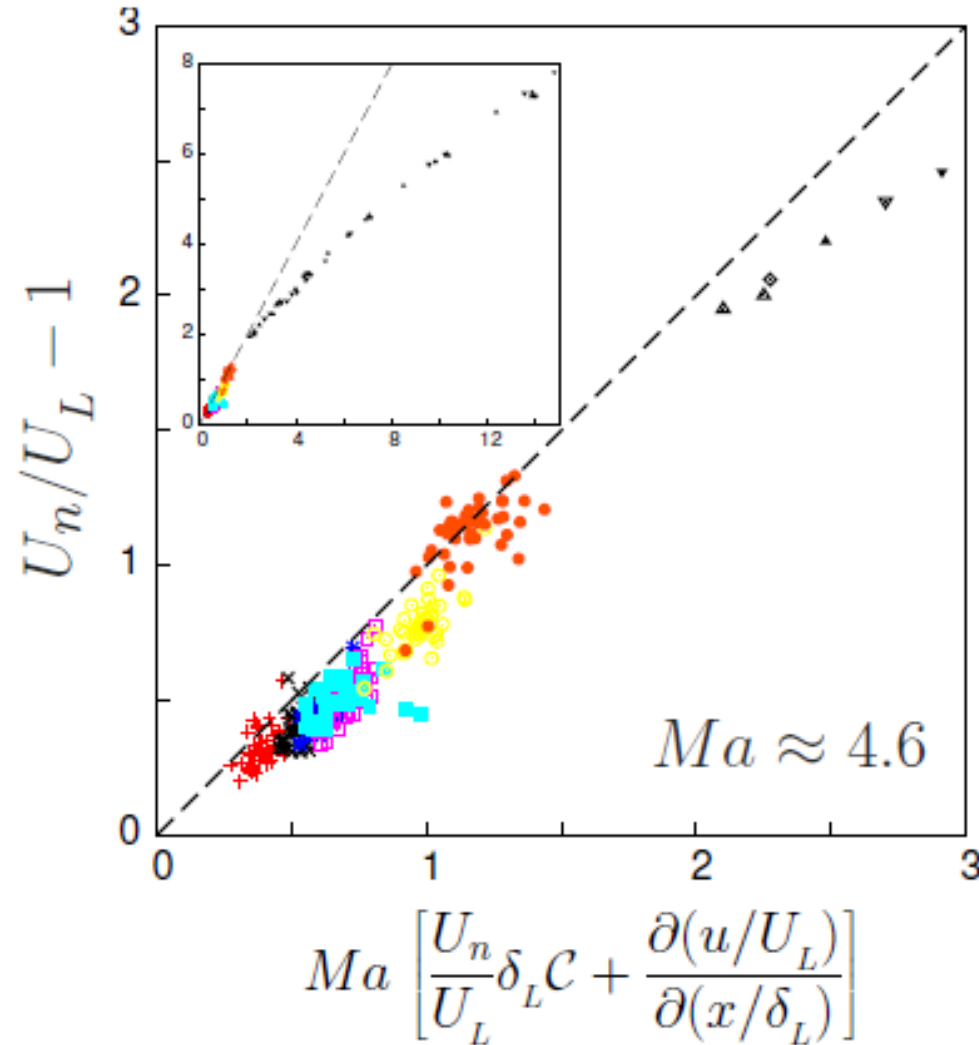
$U_n / UI-1$ vs. S/UI – Linear correlation (flame tip, $\phi = 1.40$)



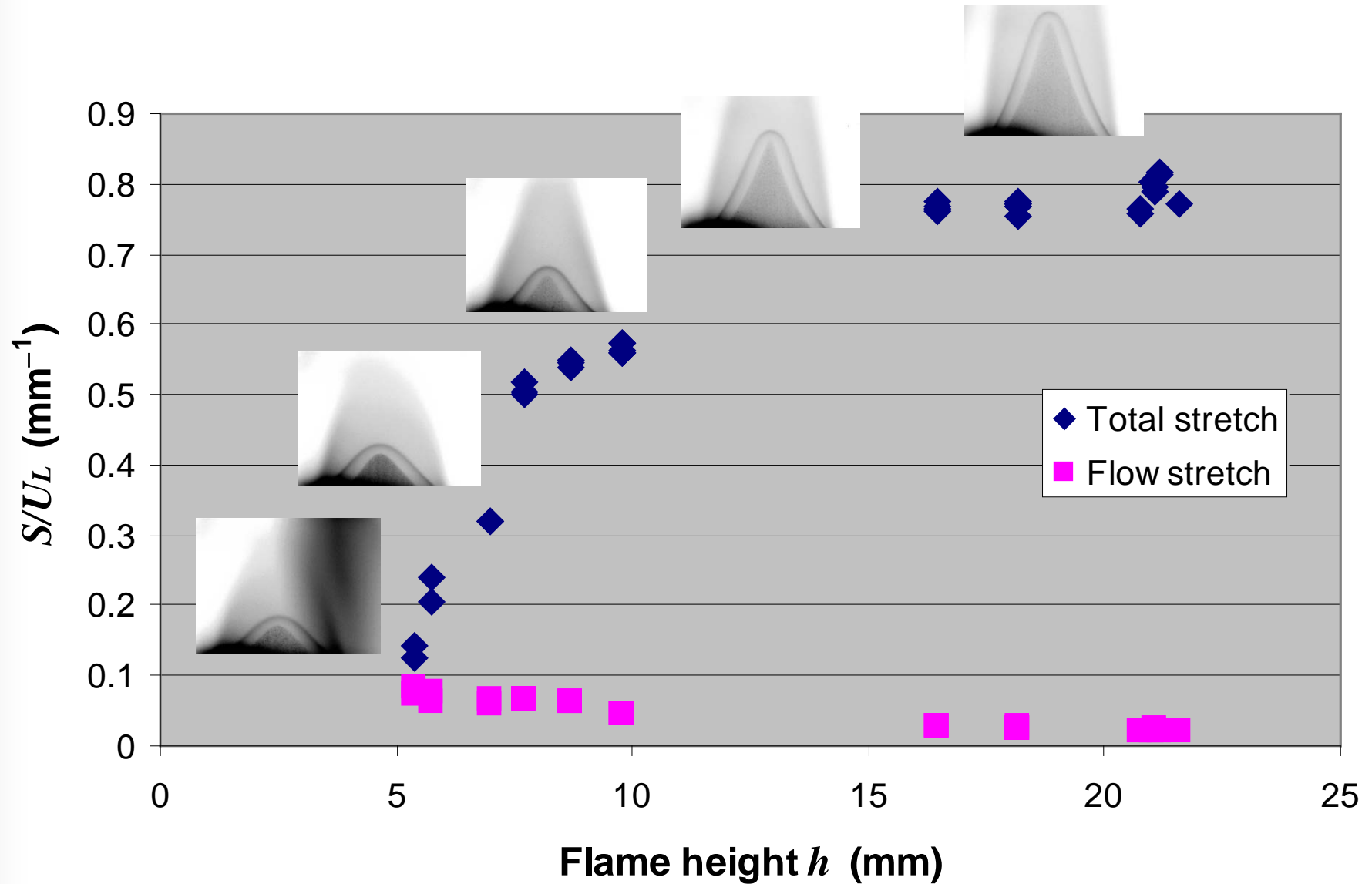
$(U_n/U_L - 1)$ vs. S/U_L (flame tip, $\phi = 1.43$)



EXTENDED MARKSTEIN RELATION



γ
$\triangle = 3$
$\nabla = 5$
$\circ = 6$
$\square = 7$
Color = Experiments



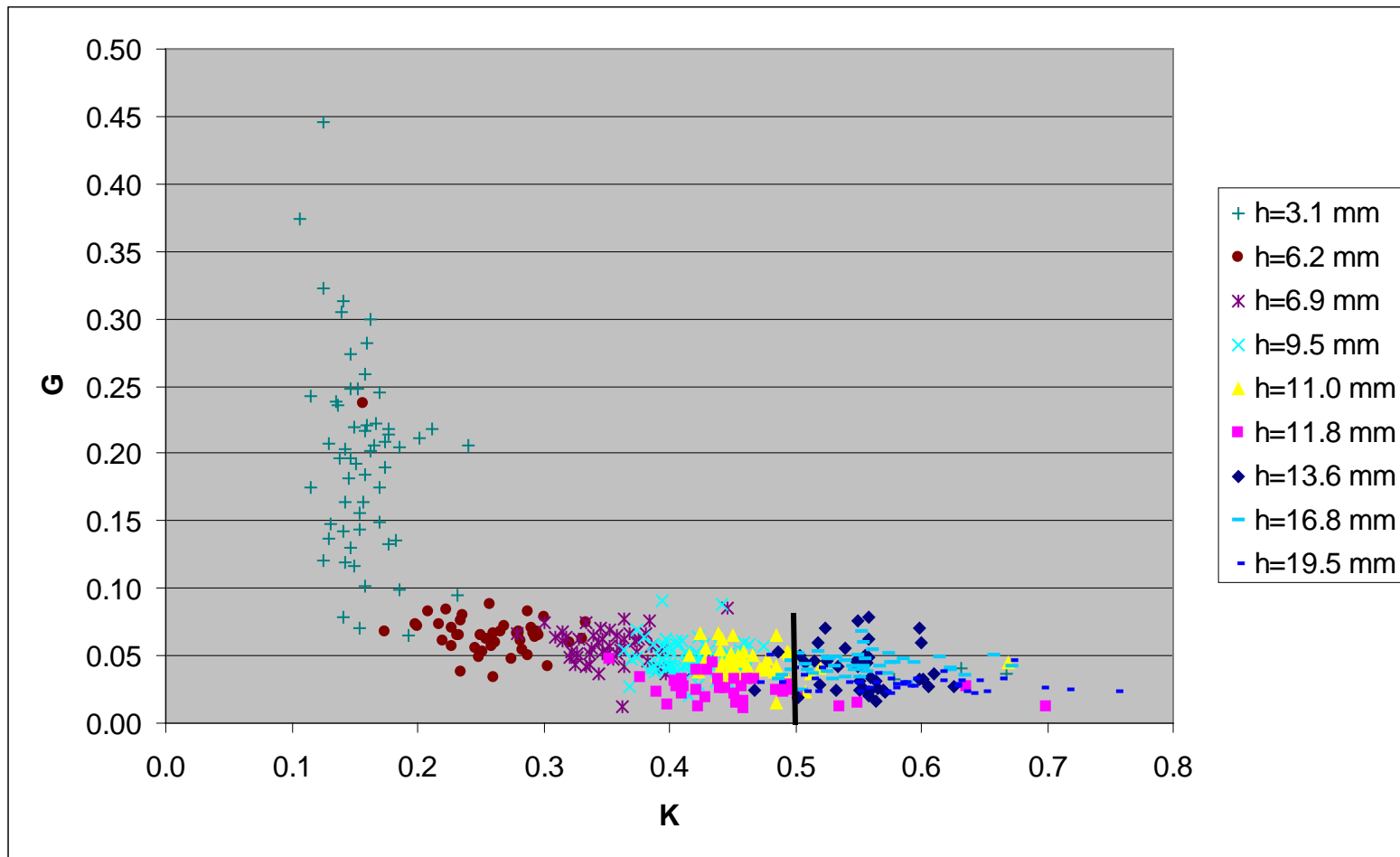
BURNING VELOCITY vs. CURVATURE & STRAIN RATE

(flame tip, $\phi = 1.43$)

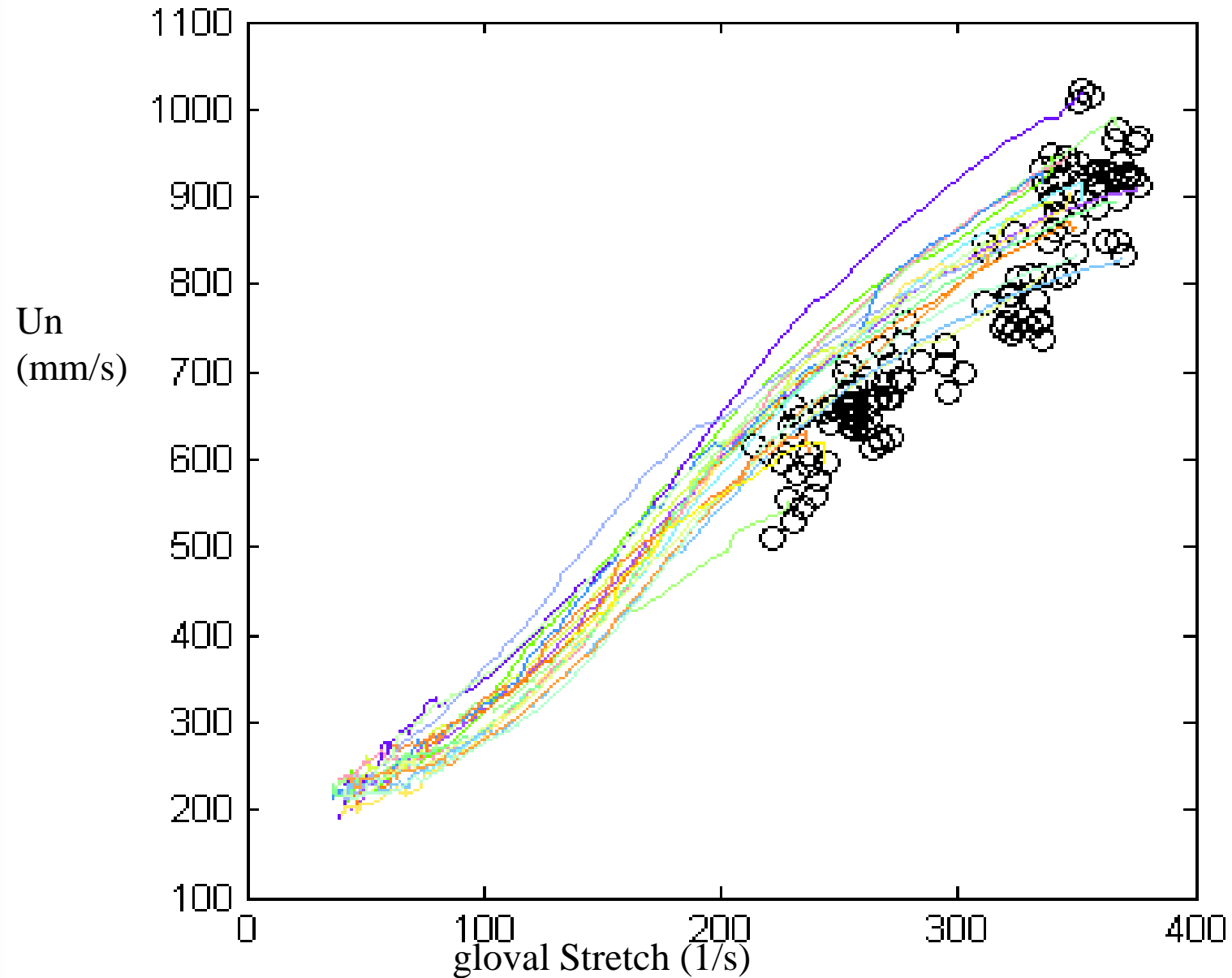


Two-variable regression:

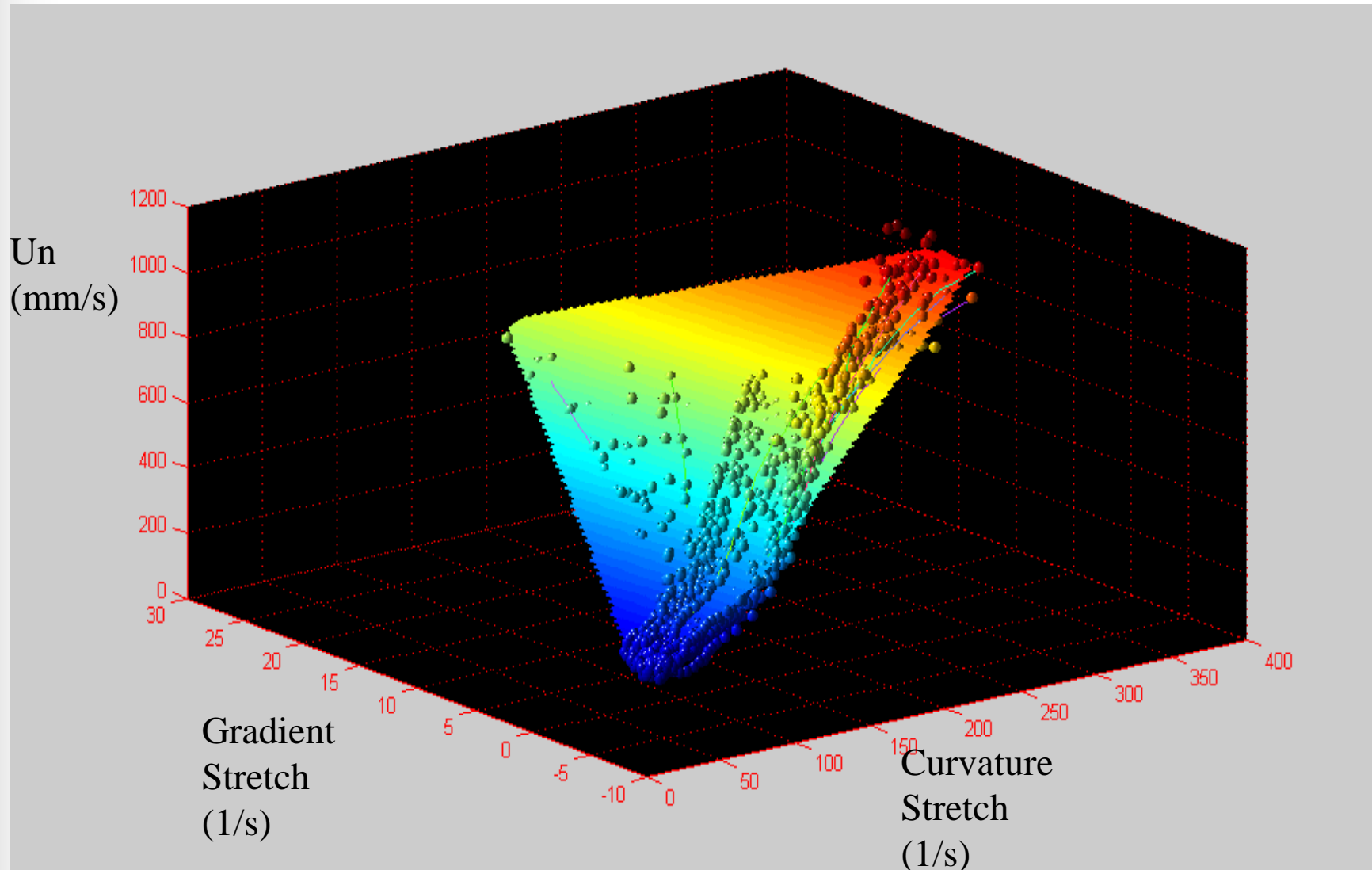
$$\frac{U_n}{U_L} - 1 = \mathcal{L}_K (\nabla \cdot \mathbf{n}) + \mathcal{L}_G \left(\frac{1}{U_L} \mathbf{n} \cdot \nabla \mathbf{u} \cdot \mathbf{n} \right)$$



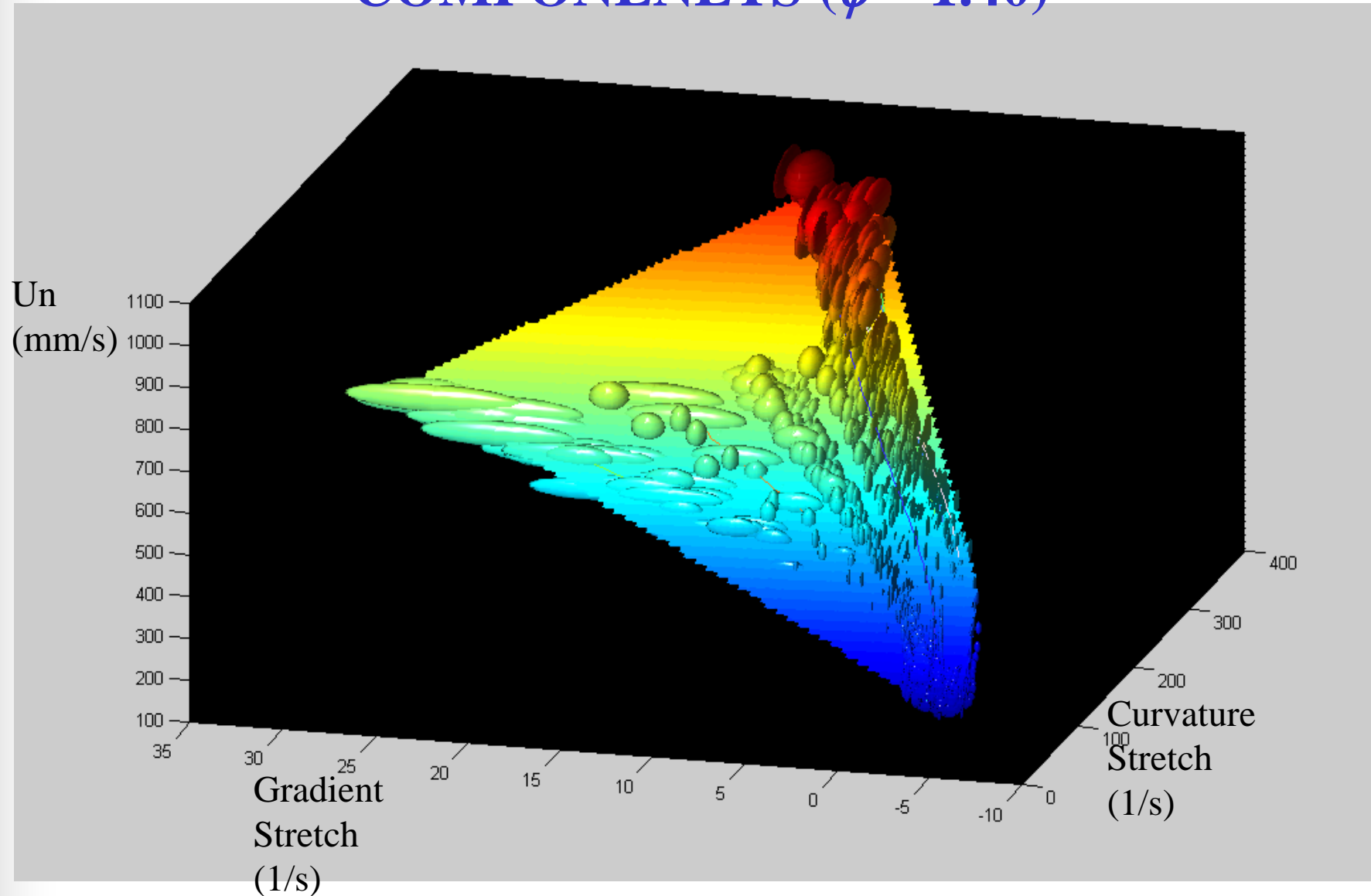
BURNING VELOCITY v_s vs. FLAME STRETCH ($\phi = 1.40$)



BURNING VELOCITY *vs.* STRETCH COMPONENTS ($\phi = 1.40$)



BURNING VELOCITY v_s vs. STRETCH COMPONENTS ($\phi = 1.40$)



CONTENT

- Motivation.
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- Discussion of results
- **Conclusions.**

CONCLUSIONS



- A PIV-based system has been set-up for the simultaneous measurement of the local burning velocity of premixed flames and the flame stretch due to the flame front curvature and the incoming flow strain rate.
- In Bunsen flame tips, these measurements allow the indirect determination of the Markstein length, according to the linear theory (Clavin & Joulin, 1983).
- The experimental results confirm the existence of two different values of the Markstein length when the flame strain rate becomes large. However, one single value of the Markstein length remains even for moderate values the flame curvature. The linear relation becomes less accurate when the stretch is very large, and a break down is observed, probably related to the transition rounded-tip to slender-tip.