On the Combustion of Hollow Cone Sprays Generated by Pressure-Swirl Injectors

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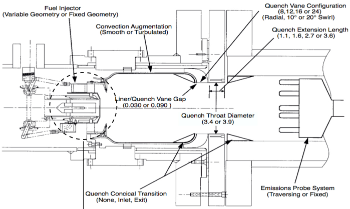
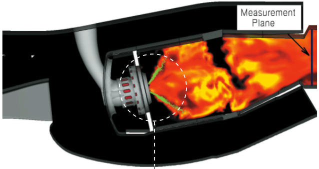
Liquid-Fuel Injection in Gas-Turbine Combustors

RQL COMBUSTOR CONCEPT (NASA)



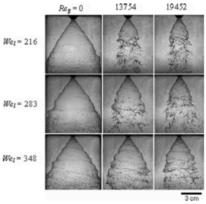
APTE & MOIN (2006), LES – LAGRANGE

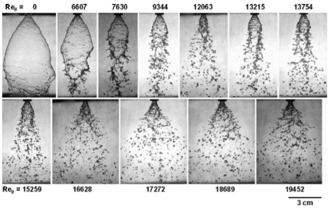


→ COMPLEX FUEL–ATOMIZATION PHYSICS (THIN FILMS, SWIRL, TURBULENT COFLOWS)

→ NUMERICAL SIMULATIONS OF THESE PROCESSES ARE COSTLY AND INACCURATE
QUANTITATIVE EXPERIMENTS ARE SCARCE BECAUSE OF HIGH LIQUID–VOLUME FRACTION

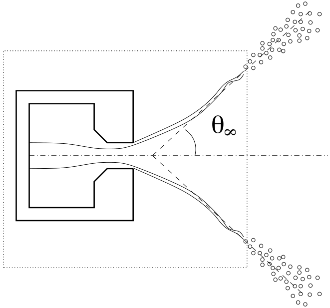
→ **NEED ANALYTICAL MODELS OF INJECTION BASED ON CONSERVATION LAWS**

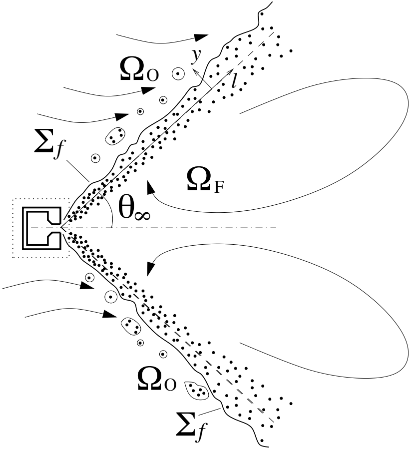




KULKARNI et al. (2010)

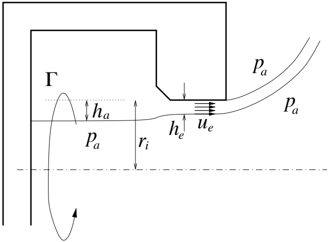
Hollow Cone Sprays Generated by Pressure-Swirl Injectors





Taylor Analysis of the inviscid swirl atomizer (1948)

The liquid fuel is forced through tangential slots to generate a strong swirling flow in the atomizer chamber.



Q volumetric flow rate
Γ : circulation of the azimuthal velocity
r_i : injector radius

Inviscid Axisymmetric Flow
$\bar{v} \cdot \nabla(rw) = 0 \longrightarrow w = \frac{\Gamma}{2\pi r}$

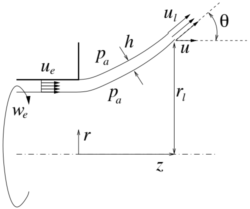
Conservation of Head: $\frac{p_a}{\rho} + \frac{1}{2}\left(\frac{\Gamma/(2\pi)}{r_i - h_a}\right)^2 = \frac{p_a}{\rho} + \frac{1}{2}\left(\frac{\Gamma/(2\pi)}{r_i - h_e}\right)^2 + \frac{u_e^2}{2}$

$h_a \sim h_e \ll r_i \longrightarrow \boxed{Q = 2\pi r_i h_e u_e = \frac{\Gamma}{r_i^{1/2}}[2h_e^2(h_a - h_e)]^{1/2}}$

Q is maximum if $h_e = \frac{2}{3}h_a$, that is if $u_e = \frac{\Gamma}{2\pi r_i}\sqrt{\frac{h_e}{r_i}} = w_e\sqrt{\frac{h_e}{r_i}}$

$$\boxed{h_e = \left(\frac{Q}{\Gamma}\right)^{2/3} r_i^{1/3}} \quad \boxed{w_e = \frac{\Gamma}{2\pi r_i}} \quad \boxed{u_e = \frac{Q^{1/3}\Gamma^{2/3}}{2\pi r_i^{4/3}}}$$

Steady Axisymmetric Thin Films



$$\boxed{\mathcal{S} = \frac{w_e}{u_e} = \left(\frac{\Gamma r_i}{Q}\right)^{1/3} = \sqrt{\frac{r_i}{h_e}} \gtrsim 1}$$

Conservation of circulation

$$w_l = \frac{\Gamma}{2\pi r_l} \longrightarrow \boxed{\frac{w_l}{w_e} = \frac{r_i}{r_l}}$$

Conservation of total pressure head:

$$u_l^2 + w_l^2 = u_e^2 + w_e^2 \longrightarrow \boxed{\left(\frac{u_l}{u_e}\right)^2 = 1 + \mathcal{S}^2\left[1 - \left(\frac{r_i}{r_l}\right)^2\right]}$$

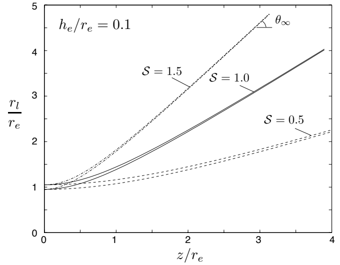
Conservation of axial momemtum flux $\Rightarrow u = u_l \cos\theta = u_e$

$$\frac{u_l}{u_e} = \frac{1}{\cos\theta} = \sqrt{\left(\frac{\mathrm{d}r_l}{\mathrm{d}z}\right)^2 + 1} \longrightarrow \boxed{\frac{\mathrm{d}r_l}{\mathrm{d}z} = \mathcal{S}\sqrt{1 - \left(\frac{r_i}{r_l}\right)^2}}$$

As $r_l \to \infty$ the swirling motion decays (i.e., $w_l \to 0$), so that

$$\frac{u_l}{u_e} \longrightarrow \boxed{\frac{u_{l\infty}}{u_e} = \sqrt{1 + \mathcal{S}^2}} \text{ and } \tan\theta = \frac{\mathrm{d}r_l}{\mathrm{d}z} \longrightarrow \boxed{\tan\theta_\infty = \mathcal{S}}$$

Steady Axisymmetric Thin Films



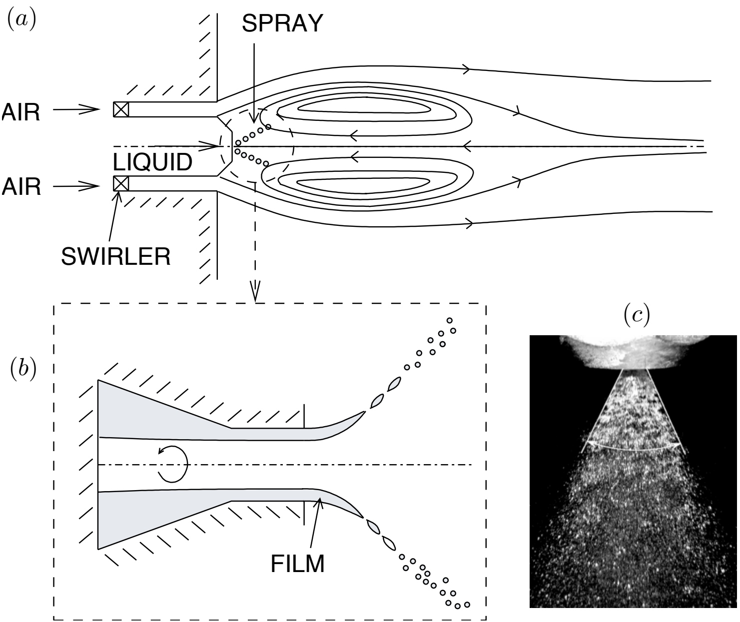
$$\frac{dr_l}{dz} = \mathcal{S}\sqrt{1 - \left(\frac{r_i}{r_l}\right)^2}, \; r_l(0) = r_i$$

$$\boxed{\frac{r_l}{r_i} = \sqrt{1 + \left(\frac{\mathcal{S}z}{r_i}\right)^2}}$$

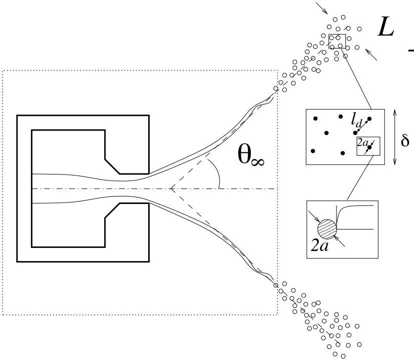
$$\frac{u_l}{u_e} = \sqrt{1 + \mathcal{S}^2\left[1 - \left(\frac{r_i}{r_l}\right)^2\right]} \longrightarrow \boxed{\frac{u_l}{u_e} = \frac{\sqrt{1+(\mathcal{S}^2+1)\left(\frac{\mathcal{S}z}{r_i}\right)^2}}{\sqrt{1+\left(\frac{\mathcal{S}z}{r_i}\right)^2}}}$$

$$Q = 2\pi r_i h_e u_e = 2\pi r_l h u_l \longrightarrow \boxed{\frac{h}{h_e} = \frac{r_i u_e}{r_l u_l} = \frac{1}{\sqrt{1+(\mathcal{S}^2+1)\left(\frac{\mathcal{S}z}{r_i}\right)^2}}}$$

Liquid-Film Break-Up and Atomization



The Two-Continua Description of Spray Combustion



Typical spray-combustion applications

$$L \gg l_d \sim n^{-1/3}$$

Under local stoichiometric conditions

$$\rho_l a^3 \sim S \rho_g l_d^3$$

$$\frac{a}{l_d} \sim \left(\frac{\rho_g}{S\rho_l}\right)^{1/3} \sim 0.1 \ll 1$$

A continuum description of the liquid phase can be carried out for all computational cells with size δ such that

$$L \gg \delta \gg l_d \gg a$$

The Two-Continua Description of Spray Combustion

The chemistry is described in terms of an overall irreversible reaction

$$\mathbf{F} + s\mathbf{O_2} \rightarrow (1+s)\mathbf{P} + q \qquad S = s/Y_{\mathrm{O_2}_{air}} \simeq 15$$

Gas-Phase Equations for a monodisperse spray ($\hat{Y}_{\mathrm{O}} = \frac{Y_{\mathrm{O_2}}}{Y_{\mathrm{O_2}_{air}}}$):

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \bar{v}) = n\dot{m}$$

$$\frac{\partial}{\partial t}(\rho \bar{v}) + \nabla \cdot (\rho \bar{v} \bar{v}) = \nabla \cdot \bar{\bar{\tau}} - \nabla p + n\dot{m}\bar{v}_d - n\bar{f}$$

$$\frac{\partial}{\partial t}(\rho Y_{\mathrm{F}}) + \nabla \cdot (\rho \bar{v} Y_{\mathrm{F}}) - \nabla \cdot \left(\frac{\rho D_T}{L_{\mathrm{F}}} \nabla Y_{\mathrm{F}}\right) = \omega_{\mathrm{F}} + n\dot{m}$$

$$\frac{\partial}{\partial t}\left(\rho \hat{Y}_{\mathrm{O}}\right) + \nabla \cdot \left(\rho \bar{v} \hat{Y}_{\mathrm{O}}\right) - \nabla \cdot \left(\frac{\rho D_T}{L_{\mathrm{O}}} \nabla \hat{Y}_{\mathrm{O}}\right) = S\omega_{\mathrm{F}}$$

$$\frac{\partial}{\partial t}(\rho Y_{\mathrm{P}}) + \nabla \cdot (\rho \bar{v} Y_{\mathrm{P}}) - \nabla \cdot \left(\frac{\rho D_T}{L_{\mathrm{P}}} \nabla Y_{\mathrm{P}}\right) = (1+S)\,\omega_{\mathrm{F}}$$

$$\frac{\partial}{\partial t}(\rho c_p T) + \nabla \cdot (\rho \bar{v} c_p T) - \nabla \cdot (\rho D_T c_p \nabla T) = -q\omega_{\mathrm{F}}$$

$$-n\left[\dot{m}\left(L_v - c_p T_d\right) + \dot{q}_d\right] + \frac{\partial p}{\partial t}$$

The Two-Continua Description of Spray Combustion

Liquid-Phase Equations:

$$\begin{aligned}
\frac{\partial n}{\partial t} + \nabla \cdot (n \bar{v}_d) &= 0 \\
\frac{4}{3}\pi \rho_l a^3 c_l \left(\frac{\partial T_d}{\partial t} + \bar{v}_d \cdot \nabla T_d \right) &= \dot{q}_d \\
\frac{\partial}{\partial t}\left(\frac{4}{3}\pi \rho_l a^3 \right) + \bar{v}_d \cdot \nabla \left(\frac{4}{3}\pi \rho_l a^3 \right) &= -\dot{m} \\
\frac{4}{3}\pi \rho_l a^3 \left(\frac{\partial \bar{v}_d}{\partial t} + \bar{v}_d \cdot \nabla \bar{v}_d \right) &= \bar{f}
\end{aligned}$$

Equation of State: $\frac{p}{\rho} = R^0 T \sum_i \frac{Y_i}{M_i}$

The Two-Continua Description of Spray Combustion

Source terms $\bar{f}$, $\dot{m}$, and $\dot{q}_d$

- Under local stoichiometric conditions

$$\rho_l a^3 \sim S\rho_g l_d^3 \quad \rightarrow \quad \frac{a}{l_d} \sim \left(\frac{\rho_g}{S\rho_l}\right)^{1/3} \sim 0.1 \ll 1$$

- Isolated droplet response in the local gas environment, which is created collectively by all droplets.
- The droplet response depends on the local Reynolds number $Re = 2aU_r/\nu_g$ based on the relative velocity $U_r = |\bar{v} - \bar{v}_d|$.

For $Re \ll 1$:

$$\bar{f} = 6\pi\mu a(\bar{v} - \bar{v}_d)$$

If $\frac{L_v}{R_F T_B} \simeq 15 \gg 1$ and $t_{c,d} = \frac{a^2}{\alpha_l} \ll t_{v,d} \sim \frac{a^2}{\alpha_g}\frac{\rho_l}{\rho_g}$:

$$\begin{cases} T_d < T_B : & \dot{q}_d = 4\pi\kappa a(T - T_d), \dot{m} = 0 \\ T_d = T_B : & \dot{q}_d = 0, \dot{m} = 4\pi\rho D_T a \ln\left[1 + \frac{c_p(T-T_B) + \hat{Y}_O q/S}{L_v}\right] \end{cases}$$

The Two-Continua Description of Spray Combustion

In the limit of infinitely fast chemistry, the reaction terms involving ω_F become Dirac-delta sinks, and can be eliminated by using linear combinations of the conservation equations to generate chemistry-free coupling functions. If unity Lewis numbers are assumed for all species $(L_F = L_O = L_P = 1)$

$$\frac{\partial}{\partial t}(\rho Y_F) + \nabla \cdot (\rho \bar{v} Y_F) - \nabla \cdot (\rho D_T \nabla Y_F) \quad = \quad \omega_F + n\dot{m}$$

and $1/S$ times

$$\frac{\partial}{\partial t}\left(\rho \hat{Y}_O\right) + \nabla \cdot \left(\rho \bar{v} \hat{Y}_O\right) - \nabla \cdot \left(\rho D_T \nabla \hat{Y}_O\right) \quad = \quad S\omega_F$$

yields

$$\frac{\partial}{\partial t}\left[\rho\left(Y_F - \frac{\hat{Y}_O}{S}\right)\right] + \nabla \cdot \left[\rho \bar{v}\left(Y_F - \frac{\hat{Y}_O}{S}\right)\right] - \nabla \cdot \left[\rho D_T \nabla\left(Y_F - \frac{\hat{Y}_O}{S}\right)\right] \quad = \quad n\dot{m}$$

This chemistry-free conservation equation is conveniently written in terms of the mixture fraction variable

$$\boxed{Z = \frac{SY_F - \hat{Y}_O + 1}{1+S}}$$

$$\frac{\partial}{\partial t}(\rho Z) + \nabla \cdot (\rho \bar{v} Z) - \nabla \cdot (\rho D_T \nabla Z) = n\dot{m}$$

The solution of this equation can be used together with the fast-chemistry assumption $Y_F \hat{Y}_O = 0$ to compute $\hat{Y}_O$ and $\hat{Y}_F$ in the region Ω_F, where $\hat{Y}_O = 0$, and in the region Ω_O, where $\hat{Y}_F = 0$, both regions being separated by the flame surface, where $\boxed{Z = Z_S = \frac{1}{1+S}}$.

This determines the reactant composition from the piecewise linear relationships

$$\begin{cases} Y_F = \frac{Z - Z_S}{1 - Z_S} \quad \text{in} \quad \Omega_F \quad \text{where} \quad Z \geq Z_s \\ \hat{Y}_O = 1 - \frac{Z}{Z_S} \quad \text{in} \quad \Omega_O \quad \text{where} \quad Z \leq Z_s \end{cases}$$

Similarly, we eliminate the reaction term from the energy conservation equation by introducing the total enthalpy

$$\boxed{H = c_p(T - T_A) + (\hat{Y}_{\mathrm{O}} - 1)q/S}$$

where T_A is the air temperature in its feed stream.

$$\begin{aligned}\frac{\partial}{\partial t}(\rho H) \quad &+ \quad \nabla \cdot (\rho \bar{v} H) - \nabla \cdot (\rho D_T \nabla H) = \\ &- \quad n\left\{\dot{m}\left[q/S + L_v - c_p\left(T_B - T_A\right)\right] + \dot{q}_d\right\}\end{aligned}$$

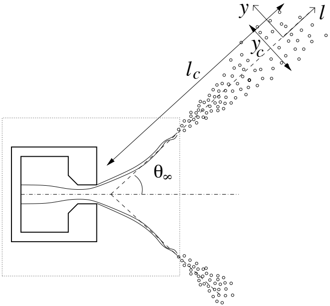
If needed, the product composition can be obtained from

$$\frac{\partial\left(\rho P\right)}{\partial t} + \nabla \cdot (\rho \bar{v} P) - \nabla \cdot (\rho D_T \nabla P) = 0$$

where

$$\boxed{P = Y_{\mathrm{P}} + (\hat{Y}_{\mathrm{O}} - 1)(1 + S)/S + Z(1 + S)/S}$$

Combustion of Hollow Cone Sprays



Cone Spray Characteristics:

Cone semiangle θ_∞

Initial droplet radius a_o

Initial droplet velocity u_{l_∞}

Droplet injection rate $\dot{n} = \frac{Q}{\frac{4}{3}\pi a_o^3}$

Droplet acceleration (evaporation) time:

$$\frac{4}{3}\pi\rho_l a^3 \frac{\mathrm{d}\bar{v}_d}{\mathrm{d}t} = 6\pi\mu_g a(\bar{v} - \bar{v}_d) \rightarrow \boxed{t_d = \frac{2}{9}\frac{\rho_l a_o^2}{\mu_g}}$$

Characteristic length scales: $\boxed{l_c = u_{l_\infty} t_d}$ and $\boxed{y_c = (\nu_g t_d)^{1/2}}$

Characteristic droplet density:

$$\dot{n} = 2\pi l_c \sin(\theta_\infty) y_c u_{l_\infty} n_c \rightarrow \boxed{n_c = \frac{\dot{n}}{2\pi \sin(\theta_\infty)\nu_g^{1/2} u_{l_\infty}^2 t_d^{3/2}}}$$

Combustion of Hollow Cone Sprays

Dimensionless variables: $l = \frac{l'}{l_c}$, $y = \frac{y'}{y_c}$, $(u, u_d) = \frac{(u', u_d')}{u_{l_\infty}}$,
$(v, v_d) = \frac{(v', v_l')}{\sqrt{\nu_g/t_d}}$, $(T, T_d) = \frac{(T', T_d')}{T_B}$, $a = \frac{a'}{a_o}$, $n = \frac{n'}{n_c}$, $H = \frac{H'}{c_p T_B}$

Steady B-L Gas-Phase Equations $\boxed{\lambda = \frac{(4/3)\pi a_o^3 \rho_l n_c}{\rho_B} \sim \frac{1}{S}}$:

$$\frac{1}{l}\frac{\partial}{\partial l}(\rho l u) + \frac{\partial}{\partial y}(\rho v) = \lambda n \dot{m}$$

$$\frac{1}{l}\frac{\partial}{\partial l}(\rho l u^2) + \frac{\partial}{\partial y}(\rho v u) = \frac{\partial^2 u}{\partial y^2} + \lambda n \dot{m} u_d + \lambda a n (u_d - u)$$

$$\frac{1}{l}\frac{\partial}{\partial l}(\rho l u Z) + \frac{\partial}{\partial y}(\rho v Z) = \frac{1}{Pr}\frac{\partial^2 Z}{\partial y^2} + \lambda n \dot{m}$$

$$\frac{1}{l}\frac{\partial}{\partial l}(\rho l u H) + \frac{\partial}{\partial y}(\rho v H) = \frac{1}{Pr}\frac{\partial^2 H}{\partial y^2} - \lambda n (\gamma \dot{m} + \dot{q}_d)$$

with $\gamma = \frac{q/S + L_v - c_p(T_B - T_A)}{c_p T_B}$ and $\dot{m} = 0$ and $\dot{q}_d = \frac{2a}{3Pr}(T - T_d)$ if $T_d < 1$ and $\dot{m} = \frac{2a}{3Pr}\ln\left(1 + \frac{T - 1 + \hat{Y}_O \bar{q}/S}{\beta}\right)$ and $\dot{q}_d = 0$ if $T_d = 1$.

Liquid-Phase Equations:

$$\frac{1}{l}\frac{\partial}{\partial l}(nlu_d) + \frac{\partial}{\partial y}(nv_d) = 0$$

$$u_d\frac{\partial}{\partial l}(a^3T_d) + v_d\frac{\partial}{\partial y}(a^3T_d) = \frac{c_p}{c_l}\dot{q}_d \begin{cases} = \frac{2a}{3Pr}(T - T_d) \text{ if } T_d < 1 \\ = 0 \text{ if } T_d = 1 \end{cases}$$

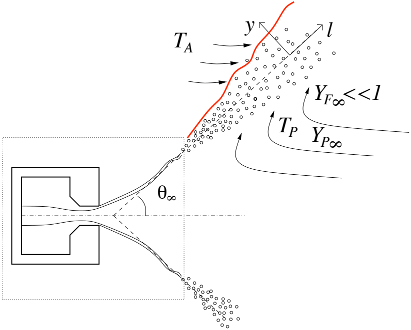
$$u_d\frac{\partial a^3}{\partial l} + v_d\frac{\partial a^3}{\partial y} = -\dot{m} \begin{cases} = 0 \text{ if } T_d < 1 \\ = \frac{2a}{3Pr}\ln\left(1 + \frac{T - 1 + \hat{Y}_O\bar{q}/S}{\beta}\right) \text{ if } T_d = 1 \end{cases}$$

$$u_d\frac{\partial u_d}{\partial l} + v_d\frac{\partial u_d}{\partial y} = \frac{1}{a^2}(u - u_d)$$

$$u_d\frac{\partial v_d}{\partial l} + v_d\frac{\partial v_d}{\partial y} = \frac{1}{a^2}(v - v_d)$$

Combustion of Hollow Cone Sprays

Boundary conditions $Z = \frac{SY_F - \hat{Y}_O + 1}{1+S}$, $H = (T - T_A) + \frac{\bar{q}(\hat{Y}_O - 1)}{S}$



Oxidizer side ($y \to +\infty$):
$u = Z = H = 0$

Product side ($y \to -\infty$):
$u = Z - Z_P = H - H_P = 0$

where
$Z_P = \frac{SY_{F\infty} + 1}{1+S} > Z_s$
and
$H_P = T_P - T_A - \frac{\bar{q}}{S}$

Integral conservation equations:

$$l \int_{-\infty}^{+\infty} (n u_d) \mathrm{d}y = 1 \quad \text{and} \quad l \int_{-\infty}^{+\infty} (\lambda^{-1} \rho u^2 + n a^3 u_d^2) \mathrm{d}y = 1$$

Note that as $l \to 0$: $u_d \to 1 \Rightarrow \boxed{n \sim \frac{1}{yl} \to \infty}$

Combustion of Hollow Cone Sprays

As $l \longrightarrow 0$ there exists a self-similar solution with two regions

- Inner spray ($y \sim l$):

$$\xi = \frac{y}{l}\left\{\begin{array}{l} a \longrightarrow 1 \\ u_d \longrightarrow 1 \\ T_d \longrightarrow T_{d_0} \end{array}\right\}, n = \frac{N(\xi)}{l^2}, v_d = \xi, \left\{\begin{array}{l} T = T_d \\ u = 1 \end{array}\right.$$

- Surrounding gas flow with $n = 0$ ($y \sim l^{1/2}$):

$$\eta = \frac{y}{\sqrt{l}},\ \rho u = F_\eta,\ \rho v = (\tfrac{1}{2}\eta F_\eta - \tfrac{3}{2}F)$$

$$\left(\frac{F_\eta}{\rho}\right)_{\eta\eta} + \frac{3}{2}F\left(\frac{F_\eta}{\rho}\right)_\eta = Z_{\eta\eta} + \frac{3}{2}PrFZ_\eta = H_{\eta\eta} + \frac{3}{2}PrFH_\eta = 0$$

$$\left\{\begin{array}{ll} \eta \longrightarrow \infty: & F_\eta = Z = H = 0 \\ \eta \longrightarrow -\infty: & F_\eta = Z - Z_P = H - H_P = 0 \\ & F = 0 \quad (v = 0) \\ \eta = 0: & F_\eta = T_o \quad (u = u_d = 1) \\ & H = T_o - T_A + (\hat{Y}_O - 1)\bar{q}/S \quad (T = T_d = T_{d_0}) \end{array}\right.$$

Combustion of Hollow Cone Sprays

Initial profiles

$$T_P = 1.5,\ Y_{F\infty} = 0.2,\ T_A = 0.8,\ T_{d_0} = 0.6,\ Z_s = 1/15.3$$