Numerical Simulation and Reduced Order Modelling of Steel Products Heating in Industrial Furnaces.

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Outline

1. Motivation and goals
   - Heat treatment of forged axles
   - Reheating furnace in a hot rolling plant
2. Heat treatment of forged axles
   - A 2D quasi–steady model
   - Fast direct and inverse design
3. Reheating furnace in a hot rolling plant
   - Reduced order model
   - Implementation and preliminary results
4. Concluding remarks

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Forged (automotive) axles manufacturing

(Usual) Stages in manufacturing process

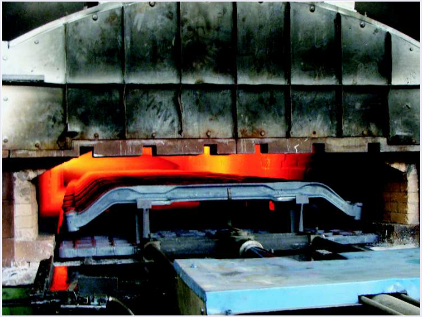
(1) hot forging
- material heating furnace
- forging press (hydraulic hammer)

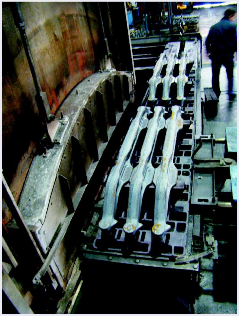
(2) heat treatment I: quench hardening
- **austenizing**
- quenching

(3) heat treatment II: tempering
- reheating
- controlled (ambient) cooling

heat treatment I: quench hardening

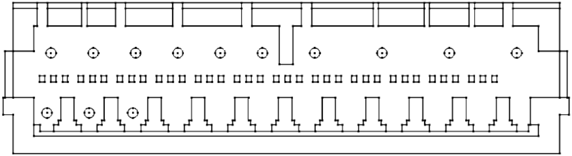
Austenizing furnace

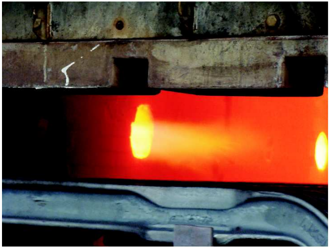




CIE Automotive

Austenizing furnace





Goal: analysis of furnace configuration and operation conditions

Design of furnace (re)configuration

- Prediction of steel pieces heating for a given furnace design (and operation conditions) in the framework of the modification of an existing furnace

Design of operation conditions

- Direct design: prediction of axles heating for given operation conditions
- Inverse design: determine optimal operation conditions (minimizing some objetive function)

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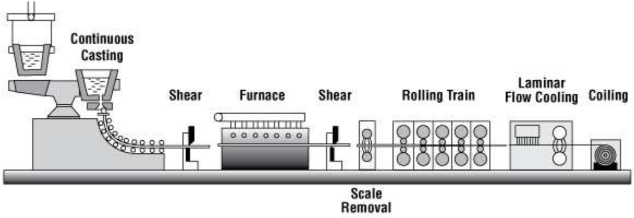
Wire rod and corrugated manufacturing

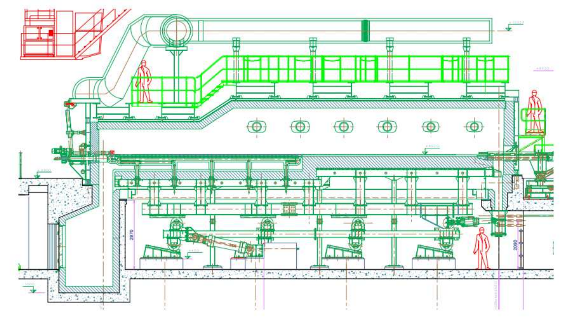
Stages in manufacturing process

(1) material (billets) reheating
(2) hot rolling (rod mills)
(3) cooling in water boxes



Reheating furnace

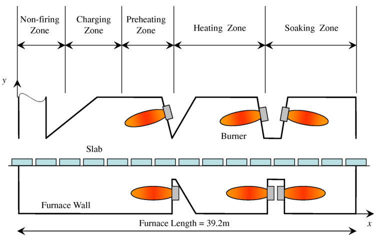
Walking beam (billet reheating) furnace

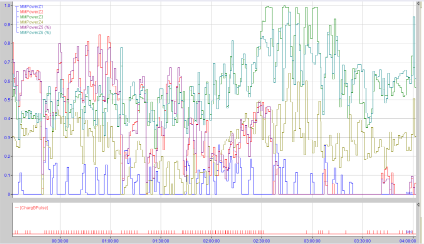


RUSSULA

Reheating furnace (cont.)

Billet reheating furnace operation





Control strategy

- preset heating curves (set points)
- event–related corrections (recipes)

Goal: fast tool for billet heating prediction

Model based control of reheating furnace

- billet heating prediction under dynamical conditions
  - time–dependent power of (group of) burners
  - non–steady operation of walking beam system (resulting in variable residence times)
  - non–regular billets feeding (presence of *gaps*)
- *real time* implementation of a simulation tool
  - to be used in control strategy

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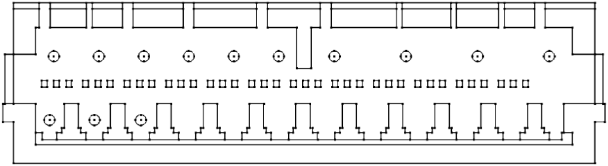
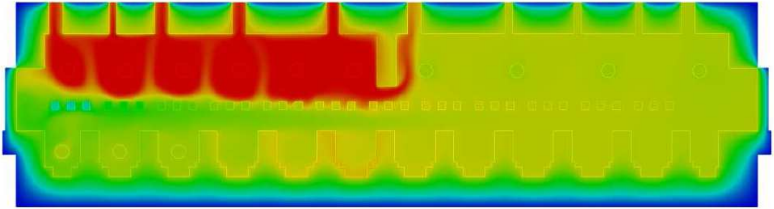
Basic aspects of the model



Involved phenomena

- combustion in burners
- thermofluid dynamics of combustion products
- heat transfer in pieces and furnace walls
- thermal radiation in furnace chamber

Basic aspects of the model (cont.)



Simplifying hypotheses

- complete combustion on a known flame surface
- gases (except flame) not participating in thermal radiation
- steady temperatures on furnace walls and gases
- 2D model (over mean vertical section)

Derivation of a global model: segregated approach

Submodel 1: furnace walls

- Thermal radiation: refractory–flames–axles
- Convection from/to gases (chamber and ambient)

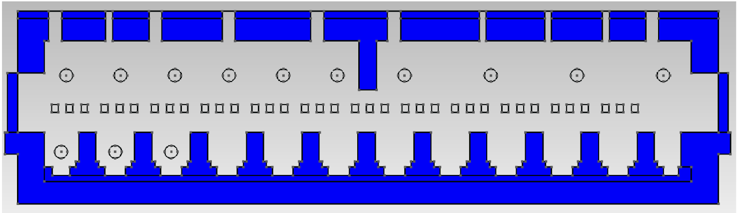
Submodel 2: thermofluid dynamics of combustion products

- Convection from/to furnace walls and axles

Submodel 3: axles heating

- Evolution problem
- Thermal radiation and convection from/to gases

Furnace walls submodel



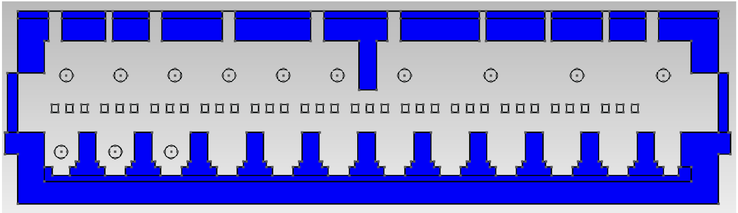
On furnace walls

$$-\mathrm{div}(k_e \vec{\nabla} T_e) = 0$$

Boundary conditions

- Outer boundary: $-k_e \frac{\partial T_e}{\partial n} = h(T_e - T_\infty)$
- Inner boundary: $-k_e \frac{\partial T_e}{\partial n} = q_{conv} + q_{rad}$

Furnace walls submodel (II)



Surface–to–surface thermal radiation

For k–th surface element:

$$q_{in}^{k} = \frac{1}{A_k} \sum_{j=1}^{N_{rad}} A_j F_{jk} q_{out}^{j}$$

$$q_{out}^{k} = \rho_k q_{in}^{k} + \epsilon_k \sigma T_k^4$$

Furnace walls submodel (III)

Flux computation on participating surfaces: Gebhardt factors

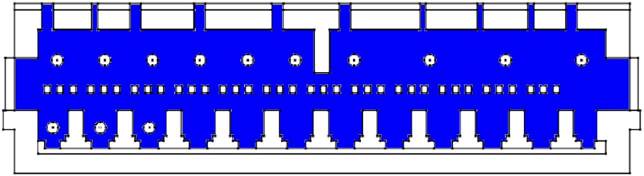
$$q^k_{out} - \rho_k \sum_{j=1}^{N_{rad}} F_{jk} q^j_{out} = \epsilon_k \sigma T_k^4$$

$$q^k_{net} = \sigma \epsilon_k T_k^4 - \frac{\sigma \epsilon_k}{A_k} \sum_{j=1}^{N_{rad}} G_{jk} T_j^4$$

Conditions on burners: energy balance on flame surface

$$\dot{W}_{burner} = \sum_{k=1}^{N_{burner}} (q^k_{net} + \rho \Delta h \vec{v} \cdot \vec{n} A_k)$$

Thermofluid dynamics submodel



Turbulent, steady, compressible flow

$$\mathrm{div}(\rho_g \vec{U}) = 0$$

$$\mathrm{div}(\rho_g \vec{U} \otimes \vec{U}) + \vec{\nabla} P - \mathrm{div}(\mu(\vec{\nabla}\vec{U} + (\vec{\nabla}\vec{U})^T) = \mathrm{div}\tau^R$$

$$\mathrm{div}(\rho_g \vec{U} T_g) - \mathrm{div}((k + k_T)\nabla T_g) = 0$$

Thermofluid dynamics submodel (II)

Turbulence model: standard $k - \epsilon$

$$\mathrm{div}(\rho_g \vec{U} k) - \mathrm{div}((\mu + \frac{\mu_T}{\sigma_k})\vec{\nabla} k) = \tau^R : \vec{\nabla}\vec{U} - \rho_g \epsilon$$

$$\rho_g \vec{U} \cdot \vec{\nabla}\epsilon - \mathrm{div}((\mu + \frac{\mu_T}{\sigma_\epsilon})\vec{\nabla}\epsilon) = C_{\epsilon 1}\frac{\epsilon}{k}\tau^R : \vec{\nabla}\vec{U} - C_{\epsilon 2}\rho_g \frac{\epsilon^2}{k}\rho_g \epsilon$$

Boundary conditions

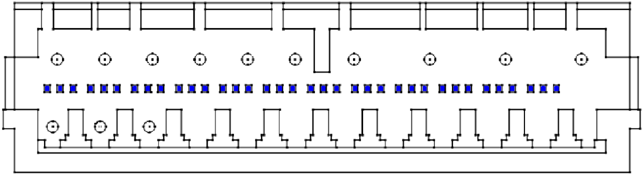
Wall laws on:

- refractory
- axles

Inlet conditions (known velocity and temperature) on:

- flame surfaces

Axles heating submodel



Axles heating

$$\rho_p c_p \frac{\partial T_p}{\partial t} - \mathrm{div}(k_p \vec{\nabla} T_p) = 0$$

Boundary conditions

$$-k_p \frac{\partial T_p}{\partial n} = q_{rad} + q_{conv}$$

Axles heating submodel (II)

Initial conditions

For the *n*-th axle position, solve

$$\rho_p c_p \frac{\partial T_p^n}{\partial t} - \mathrm{div}(k_p \vec{\nabla} T_p^n) = 0 \qquad \text{with } t \in (0, t_{res})$$

and initial condition

$$T_p^n(\vec{x}, 0) = T_p^{n-m}(\vec{x}, t_{res})$$

Remarks

- *Final* values are used in submodels coupling
- heat flux on boundary kept constant over $(0, t_{res})$

Global algorithm

Algorithm outline

- Initialize axles heating curve
- Initialize convective fluxes
- Iteration loop on submodels:
  - Solve (steady) model (radiation–conduction) on furnace
  - Solve (steady) model (thermofluid dynamics) on gases
  - Solve (evolution) model for axles heating
  - Convergence test

Remark

Some relaxation is needed to avoid numerical instabilities

Numerical discretization

Furnace (walls) submodel

- non–linearity: fixed point
- spatial discretization: $P1$ finite element
- Gebhardt (factor) matrix is stored

Gases submodel

- segregated solver (N–S, $k - \epsilon$, energy) with fixed point
- non–linearities: fixed point and Newton
- spatial discretizacion: $P2/P1$+SUPG and $P1 - b$

Axles submodel

- time integration: *BDF*–1 with time step adaption
- spatial discretizacion: $P1$ finite element

Implementation with free software tools

CAD and meshing using `Gmsh`

`http://geuz.org/gmsh/`

Model solvers using `Elmer`

`http://www.csc.fi/english/pages/elmer`

Global algorithm programming using `Python`

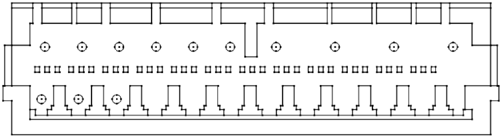
`http://www.python.org/`

`http://www.scipy.org/`

Postprocessing using `Paraview`

`http://www.paraview.org/`

Heat treatment (austenizing) furnace



Operation conditions: power

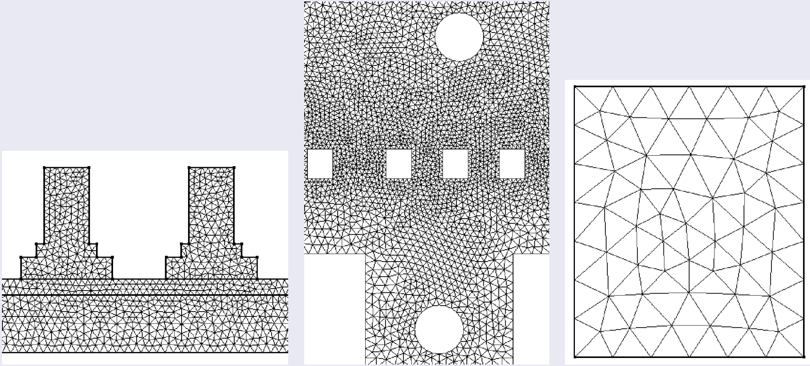
- Total furnace power: 1.81 MW
- Group I nominal power (6 burners): 1.15 MW
- Group II nominal power (7 burners): 0.66 MW

Operation conditions: feeding

- Residence time: 720 s.

Heat treatment furnace simulation

Details of meshes



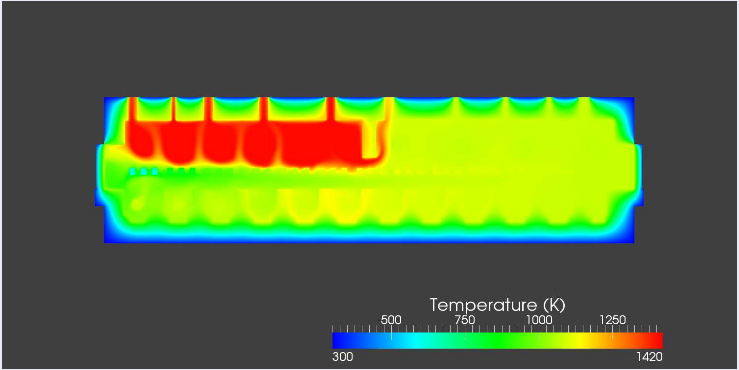
Heat treatment furnace simulation (II)

Some computational figures

- Mesh size
  - furnace (walls): 24000 nodes
  - gases: 38500 nodes
  - axles: 2300 nodes
  - radiating surfaces: 30000 edges
- Global algorithm iterations: 9
- Tolerance in convergence test: $1K$
- Computational cost:
  - 3.5 h. of computation (Core2Duo 2.5 GHz, 1 core)
  - memory peak below 1 GB

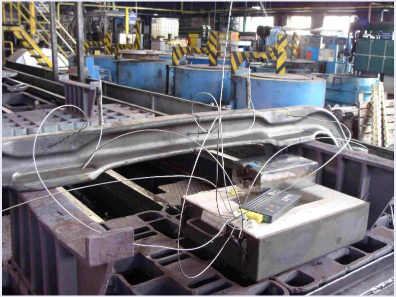
Heat treatment furnace simulation (IIO)

Numerical simulation results



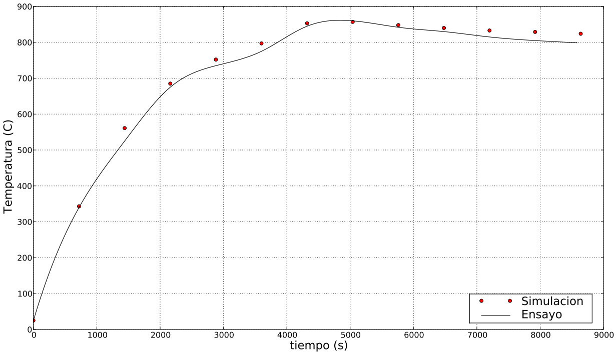
Validation

Thermocouples (Datapaq) test at CIE–Galfor



Validation (II)

Experimental and numerical results



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Fast heating prediction

Direct numerical simulation

Computation time makes non–affordable:

- integration in process simulation tools
- optimization of operation conditions

Alternative

- Preprocessing through simulation databases
- To be defined:
  - building/storage strategies
  - interpolation techniques

Simulation database storage

Storage in tensor form

Value corresponding to:

- input parameters $e_1, e_2, \ldots\ e_n$
- output parameters $s_1\ , s_2, \ldots\ s_m$

is storage as: $a_{e_1 e_2 \ldots e_n s_1 s_2 \ldots s_m}$

Simulation database storage (II)

Input parameters (e_1, e_2, ... e_n)

- feeding velocity (e_1)

  (index spans set of velocities used in database building)
- power of each group of burners (e_2, ... e_n)

  (index spans set of powers used in database building)

Output parameter (s_1)

- axle temperature

  (index spans set of points stored in database)

Singular Value Decomposition

SVD factorization

$$A = U\Sigma V^T \qquad \Sigma \text{ diagonal}, \quad U \text{ and } V \text{ orthogonal}$$

- allows compression (optimal low–rank approximations)
- gives *modal* information (on rows and columns)

Higher–Order SVD (HOSVD) factorization

$$\mathcal{A} = \mathcal{S} \times_1 U^{(1)} \times_2 U^{(2)} \cdots \times_N U^{(N)}$$

- allows compression (although $\mathcal{S}$ not diagonal)
- gives *modal* information (on each input variable!)
- (one variable) interpolation (using $U^{(j)}$ columns) to predict for new input variables

Interpolation using HOSVD

Interpolation in variable related to l–th index

$$\mathcal{P}(x^{(l)}) = \tilde{\mathcal{S}} \times_1 U^{(1)} \cdots \times_j \Pi(x^{(l)}) U^{(l)} \cdots \times_N U^{(N)}$$

$$P(x_1^{i_1}, \dots, x_l, \dots, x_N^{i_N}) =$$

$$= \sum_{j_1=1}^{N_1^T} \sum_{j_2=1}^{N_2^T} \cdots \sum_{j_N=1}^{N_N^T} s_{j_1 j_2 \dots j_N} U^{(1)}_{i_1, j_1} \dots \left(\sum_{k=1}^{N_l^T} \alpha_k(x_l) U^{(l)}_{k, j_l} \right) \dots U^{(N)}_{i_N, j_N}$$

where:

$\sum_{k=1}^{N_l^T} \alpha_k(x_l) U^{(l)}_{k,j_l}$ interpolation operator on table $\{(x_l^{i_l}, U^{(l)}_{i_l, j_l})\}_{i_l=1}^{N_l^T}$

Heating prediction using DB and HOSVD

Database building

- residence times: 540, 720 and 900 s.
- group I burners powers: 50%, 75% and 100%
- group II burners powers: 50%, 55% and 60%

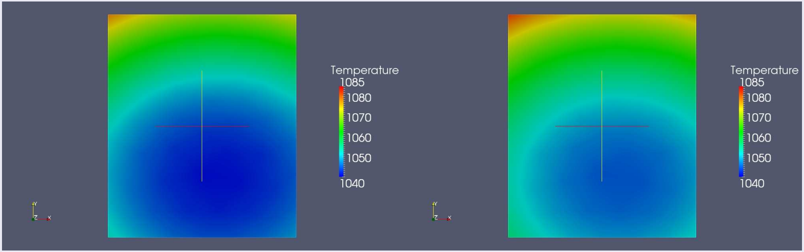
Database exploitation

- no compression used
- axle heating predicted for:
  - residence time: 630 s.
  - group I burners power: 85%
  - group II burners power: 58%
- prediction computation time $\simeq 0.1s$

Heating prediction using DB and HOSVD (II)

Axle heating prediction (central position)

Direct numerical simulation | DB/HOSVD prediction



Maximum error below 5 K.

Inverse design

Inverse design problem formulation

Minimization of a funcional based on:

- quality of heat treatment
- energy consumption
- other parameters

Solving inverse design problems

- huge cost using direct numerical simulation
- affordable cost using a HOSVD approach
- easy computation of (exact) derivatives
- (on–going) implementation of trust region techniques using a conjugate gradient (Steihaug–Toint) algorithm

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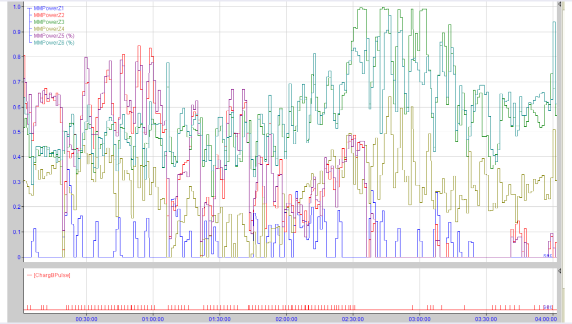
Heating prediction under dynamical conditions

Reheating furnace operation

Furnace operation slave to rolling train operation (programmed and non–programmed events)



usual variation of operation conditions (time span of 4 hours)

Billets heating modelling

(Single) Billet heating model

$$\rho c_p \frac{\partial T}{\partial t} - \mathrm{div}(k\vec{\nabla} T) = 0 \qquad \text{in } \Omega \times (t_{in}, t_{out})$$

$$T(\vec{x}, t_{in}) = T_{in} \qquad \text{in } \Omega$$

$$-k\frac{\partial T}{\partial n} = q(t) \qquad \text{on } \partial\Omega \times (t_{in}, t_{out})$$

Remarks

- $q(t)$ introduces non–local (and non–linear) behaviour
  - $q(t) = q_{conv}(t) + q_{cond}(t) + q_{rad}(t)$
  - assembling of $q(t)$ depends on (variable) billet position
- ρc_p and k are temperature–dependent

Simplified billets heating model

Reduction to a linear and local problem

Heating modelling over (t_n, t_{n+1}) with $t_{n+1} = t_n + \Delta t$:

$$\rho c_p \frac{\partial T_{n+1}}{\partial t} - \mathrm{div}(k \vec{\nabla} T_{n+1}) = 0 \qquad \text{in } \Omega \times (t_n, t_{n+1})$$

$$T_{n+1}(\vec{x}, t_n) = T_n(\vec{x}, t_n) \qquad \text{in } \Omega$$

$$-k \frac{\partial T_{n+1}}{\partial n} = q_n \qquad \text{on } \partial\Omega \times (t_n, t_{n+1})$$

with ρc_p, k and q_n computed from temperatures at t_n

Discretization with small Δt

- control of modelling error
- stability of resulting algorithm

Furnace walls heating modelling

Furnace walls heating model

$$\rho_e c_{pe} \frac{\partial T_e}{\partial t} - \mathrm{div}(k_e \vec{\nabla} T_e) = 0$$

with boundary conditions

$$-k_e \frac{\partial T_e}{\partial n} = h(T_e - T_\infty) \qquad \text{on } \Gamma_{ext}$$

$$-k_e \frac{\partial T_e}{\partial n} = q_{conv} + q_{cond} + q_{rad} \qquad \text{on } \Gamma_{int}$$

and suitable initial conditions

Reduction to a linear and local problem

- same approach used for billets heating

Heat flux (to billets and furnace walls) segregation

Flux segregation

On every boundary of billets and structure:

$$q = q_{conv} + q_{cond} + q_{rad}$$

- q_{conv}: convection heat flux (from/to gases)
- q_{cond}: conduction heat flux (through hearth)
- q_{rad}: radiation heat flux

Modelling of segregated fluxes

- q_{rad} computed from temperatures of participating surfaces
- q_{conv} and q_{cond} correlated with actual powers (so far!)

Computing radiation flux

Energy balance on each burner

- instantaneous power (could be zero) used
- temperature assumed homogeneous over each burner

$$\dot{W}_{burner} = \sum_{k=1}^{N_{burner}} (q^k_{net} + \rho \Delta h \vec{v} \cdot \vec{n} A_k)$$

Treatment of *gaps* (absence of billet charge)

- Gaps modify Gebhardt matrix (non–affordable approach)
- Instead *neutral* radiation surfaces (each gap) introduced

$$0 = \sum_{k=1}^{N_{hollow}} q^k_{net}$$

Solving heating model (billets and furnace walls)

Galerkin method

- Choice of a base $\{\Phi_i(\vec{x})\}_{i=1}^{N_{base}}$
- Find $T_h(\vec{x},t) = \sum_{i=1}^{N_{base}} \alpha_i(t)\Phi_i(\vec{x})$ such that

$$\sum_{i=1}^{N_{base}} \frac{d\alpha_i}{dt} \int_\Omega \rho c_p \Phi_i \Phi_j \, d\Omega + \sum_{i=1}^{N_{base}} \alpha_i \int_\Omega k \vec{\nabla}\Phi_i \cdot \vec{\nabla}\Phi_j \, d\Omega$$

$$= \int_{\partial\Omega} q\Phi_j \, dS \qquad \forall j = 1, 2, \ldots N_{base}$$

Matrix form

$$M\frac{d\vec{\alpha}}{dt} + K\vec{\alpha} = \vec{b}$$

A reduced order model

Alternative I: finite element method

- N_{base} = mesh degrees of freedom
- charasteristic values (number of nodes):
  - billets: 50–500 (each billet)
  - furnace walls: 10000–15000
- *real time* implementation unfeasible (or very challenging)

Alternative II: Galerkin–Proper Orthogonal Decomposition

- An *empirical* base is extracted from a set of (*snapshots*)
- N_{base} can be quite small:

  4–6 modes (furnace walls and each billet)

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Implementation of a reduced–order model (ROM)

Generation of a ROM basis

POD computation from a set of *snapshots*

- billets: heating under *nominal* steady operation
- furnace: steady temperature under different regimes

Assembling/integration of Galerkin–POD evolution equations

Preprocessing of:

- mass and stiffness *modal* matrices
- projection operator to compute modal charges
- fundamental matrices of ODE's system

Real time implementation

Dynamical model (executed every minute)

Input variables (data acquired during last minute):

- actual power of each burner (sampled every second)
- time(s) of movement of walking beam system
- time(s) of charge of new billet(s)
- temperature of charged billet(s)

Initial values:

- Mean temperature of billets (beginning of the last minute)

Output variables:

- Mean temperature of billets (end of the last minute)

Preliminary validation

Reheating furnace in (hot) rolling train

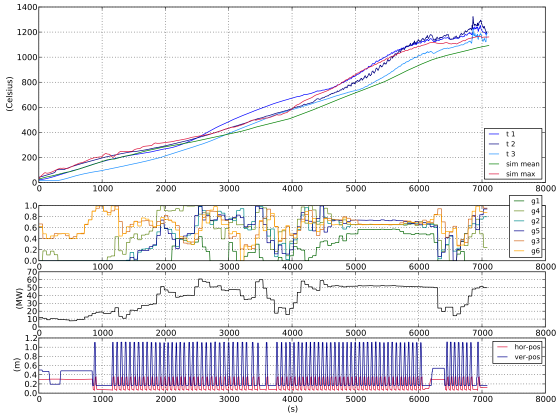
- total furnace power: 80 MW
- mean power during test I: 28 MW
- mean power during test II: 37 MW

Computational details

- computation time: 0.5–0.7 s
- used memory peak: below 65 MB

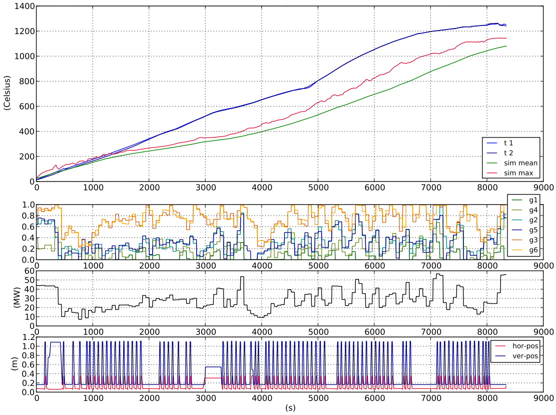
Preliminary validation: test I

Dynamical model prediction vs thermocouple signals



Preliminary validation: test II

Dynamical model prediction vs thermocouple signals



Outline

1 Motivation and goals
- Heat treatment of forged axles
- Reheating furnace in a hot rolling plant

2 Heat treatment of forged axles
- A 2D quasi–steady model
- Fast direct and inverse design

3 Reheating furnace in a hot rolling plant
- Reduced order model
- Implementation and preliminary results

4 Concluding remarks

Numerical modelling under steady conditions

Direct numerical simulation

- accurate numerical predictions
- implementation with free software tools
- fast predictions using simulation databases

Optimization of operation conditions

- affordable optimization using DB/HOSVD models

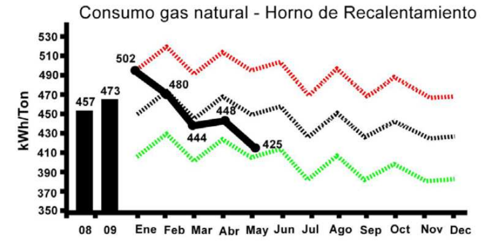
Numerical modelling under dynamical conditions

Numerical simulation tool

- implementation in a *real time* tool
- poor numerical predictions under very variable conditions

Reduction of fuel consumption

Regulation using a previous model implementation



On–going and future work

Optimization of operation conditions (under steady condition)

- Implementation of trust region methods with DB/HOSVD models

Numerical modelling under dynamical conditions

- reduced order modelling of thermofluid dynamics
- implementation of adaptive control techniques