Numerical Simulation and Reduced Order Modelling of Steel Products Heating in Industrial Furnaces.

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Workshop on mathematical modelling of combustion Santiago de Compostela, May 23 - 25, 2011
Outline

1. Motivation and goals
   - Heat treatment of forged axles
   - Reheating furnace in a hot rolling plant

2. Heat treatment of forged axles
   - A 2D quasi–steady model
   - Fast direct and inverse design

3. Reheating furnace in a hot rolling plant
   - Reduced order model
   - Implementation and preliminary results

4. Concluding remarks
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Forged (automotive) axles manufacturing

(Usual) Stages in manufacturing process

1. hot forging
   - material heating furnace
   - forging press (hydraulic hammer)
2. heat treatment I: quench hardening
   - austenizing
   - quenching
3. heat treatment II: tempering
   - reheating
   - controlled (ambient) cooling
heat treatment I: quench hardening

Austenizing furnace
Austenizing furnace
Goal: analysis of furnace configuration and operation conditions

Design of furnace (re)configuration
- Prediction of steel pieces heating for a given furnace design (and operation conditions) in the framework of the modification of an existing furnace

Design of operation conditions
- Direct design: prediction of axles heating for given operation conditions
- Inverse design: determine optimal operation conditions (minimizing some objective function)
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Wire rod and corrugated manufacturing

Stages in manufacturing process

1. material (billets) reheating
2. hot rolling (rod mills)
3. cooling in water boxes
Motivation and goals
Heat treatment of forged axles
Reheating furnace in a hot rolling plant
Concluding remarks

Reheating furnace

Walking beam (billet reheating) furnace
Reheating furnace (cont.)

Billet reheating furnace operation

Control strategy
- preset heating curves (set points)
- event–related corrections (recipes)
Goal: fast tool for billet heating prediction

Model based control of reheating furnace

- billet heating prediction under dynamical conditions
  - time–dependent power of (group of) burners
  - non–steady operation of walking beam system
    (resulting in variable residence times)
  - non–regular billets feeding (presence of gaps)

- real time implementation of a simulation tool
  - to be used in control strategy
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Basic aspects of the model

Involved phenomena

- combustion in burners
- thermofluid dynamics of combustion products
- heat transfer in pieces and furnace walls
- thermal radiation in furnace chamber
Basic aspects of the model (cont.)

Simplifying hypotheses

- complete combustion on a known flame surface
- gases (except flame) not participating in thermal radiation
- steady temperatures on furnace walls and gases
- 2D model (over mean vertical section)
Derivation of a global model: segregated approach

**Submodel 1: furnace walls**
- Thermal radiation: refractory–flames–axles
- Convection from/to gases (chamber and ambient)

**Submodel 2: thermofluid dynamics of combustion products**
- Convection from/to furnace walls and axles

**Submodel 3: axles heating**
- Evolution problem
- Thermal radiation and convection from/to gases
Furnace walls submodel

On furnace walls

\[-\text{div}(k_e \vec{\nabla} T_e) = 0\]

Boundary conditions

- Outer boundary: \(-k_e \frac{\partial T_e}{\partial n} = h(T_e - T_{\infty})\)
- Inner boundary: \(-k_e \frac{\partial T_e}{\partial n} = q_{\text{conv}} + q_{\text{rad}}\)
Furnace walls submodel (II)

Surface–to–surface thermal radiation

For \( k \)-th surface element:

\[
q_{in}^k = \frac{1}{A_k} \sum_{j=1}^{N_{rad}} A_j F_{jk} q_{out}^j
\]

\[
q_{out}^k = \rho_k q_{in}^k + \epsilon_k \sigma T_k^4
\]

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**Furnace walls submodel (III)**

Flux computation on participating surfaces: Gebhardt factors

\[
q^k_{out} - \rho_k \sum_{j=1}^{N_{rad}} F_{jk} q^j_{out} = \varepsilon_k \sigma T_k^4
\]

\[
q^k_{net} = \sigma \varepsilon_k T_k^4 - \frac{\sigma \varepsilon_k}{A_k} \sum_{j=1}^{N_{rad}} G_{jk} T_j^4
\]

Conditions on burners: energy balance on flame surface

\[
\dot{W}_{burner} = \sum_{k=1}^{N_{burner}} \left( q^k_{net} + \rho \Delta h \vec{v} \cdot \vec{n} A_k \right)
\]
Thermofluid dynamics submodel

Turbulent, steady, compressible flow

\[
\text{div}(\rho_g \vec{U}) = 0
\]

\[
\text{div}(\rho_g \vec{U} \otimes \vec{U}) + \vec{\nabla} P - \text{div}(\mu(\vec{\nabla} \vec{U} + (\vec{\nabla} \vec{U})^T)) = \text{div} \tau^R
\]

\[
\text{div}(\rho_g \vec{U}T_g) - \text{div}((k + k_T)\nabla T_g) = 0
\]
Thermofluid dynamics submodel (II)

**Turbulence model: standard \( k - \varepsilon \)**

\[
\text{div}(\rho g \vec{U} k) - \text{div}((\mu + \frac{\mu T}{\sigma_k}) \vec{\nabla} k) = \tau^R : \vec{\nabla} \vec{U} - \rho g \varepsilon
\]

\[
\rho g \vec{U} \cdot \vec{\nabla} \varepsilon - \text{div}((\mu + \frac{\mu T}{\sigma_\varepsilon}) \vec{\nabla} \varepsilon) = C_\varepsilon \frac{\varepsilon}{k} \tau^R : \vec{\nabla} \vec{U} - C_\varepsilon^2 \rho g \frac{\varepsilon^2}{k} \rho g \varepsilon
\]

**Boundary conditions**

- **Wall laws on:**
  - refractory
  - axles

- **Inlet conditions (known velocity and temperature) on:**
  - flame surfaces
Axles heating submodel

Axles heating

\[ \rho_p c_p \frac{\partial T_p}{\partial t} - \text{div}(k_p \vec{\nabla} T_p) = 0 \]

Boundary conditions

\[ -k_p \frac{\partial T_p}{\partial n} = q_{rad} + q_{conv} \]
Initial conditions

For the $n$-th axle position, solve

$$
\rho_p c_p \frac{\partial T^n_P}{\partial t} - \text{div}(k_p \vec{\nabla} T^n_P) = 0 \quad \text{with} \quad t \in (0, t_{res})
$$

and initial condition

$$
T^n_P(\vec{x}, 0) = T^{n-m}_P(\vec{x}, t_{res})
$$

Remarks

- *Final values are used in submodels coupling*
- *heat flux on boundary kept constant over $(0, t_{res})$*
Global algorithm

Algorithm outline

- Initialize axles heating curve
- Initialize convective fluxes
- Iteration loop on submodels:
  - Solve (steady) model (radiation–conduction) on furnace
  - Solve (steady) model (thermofluid dynamics) on gases
  - Solve (evolution) model for axles heating
  - Convergence test

Remark

Some relaxation is needed to avoid numerical instabilities
### Numerical discretization

#### Furnace (walls) submodel
- **non-linearity:** fixed point
- **spatial discretization:** $P1$ finite element
- Gebhardt (factor) matrix is stored

#### Gases submodel
- segregated solver ($N-S, k - \epsilon$, energy) with fixed point
- **non-linearities:** fixed point and Newton
- **spatial discretization:** $P2/P1$+SUPG and $P1 - b$

#### Axles submodel
- time integration: $BDF-1$ with time step adaptation
- **spatial discretization:** $P1$ finite element
### Implementation with free software tools

<table>
<thead>
<tr>
<th>Component</th>
<th>Software/Package</th>
<th>URL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAD and meshing</td>
<td>Gmsh</td>
<td><a href="http://geuz.org/gmsh/">http://geuz.org/gmsh/</a></td>
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Heat treatment (austenizing) furnace

Operation conditions: power
- Total furnace power: 1.81 MW
- Group I nominal power (6 burners): 1.15 MW
- Group II nominal power (7 burners): 0.66 MW

Operation conditions: feeding
- Residence time: 720 s.
Heat treatment furnace simulation

Details of meshes
Some computational figures

- **Mesh size**
  - furnace (walls): 24000 nodes
  - gases: 38500 nodes
  - axles: 2300 nodes
  - radiating surfaces: 30000 edges
- **Global algorithm iterations**: 9
- **Tolerance in convergence test**: $1K$
- **Computational cost**:
  - 3.5 h. of computation (Core2Duo 2.5 GHz, 1 core)
  - memory peak below 1 GB
Heat treatment furnace simulation (IIO)

Numerical simulation results

Temperature (K)

300 500 750 1000 1250 1420
Validation

Thermocouples (Datapaq) test at CIE–Galfor
Validation (II)

Experimental and numerical results

- Diagram showing temperature over time for simulation and experiment.
- Label axes: tiempo (s) and Temperatura (°C).
- Data points and labels: Simulacion and Ensayo.

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Fast heating prediction

Direct numerical simulation

Computation time makes non-affordable:
- integration in process simulation tools
- optimization of operation conditions

Alternative

- Preprocessing through simulation databases
- To be defined:
  - building/storage strategies
  - interpolation techniques
Simulation database storage

Storage in tensor form

Value corresponding to:
- input parameters $e_1, e_2, \ldots, e_n$
- output parameters $s_1, s_2, \ldots, s_m$

is storage as: $a_{e_1 e_2 \ldots e_n s_1 s_2 \ldots s_m}$
Simulation database storage (II)

Input parameters \((e_1, e_2, \ldots, e_n)\)
- feeding velocity \((e_1)\)
  (index spans set of velocities used in database building)
- power of each group of burners \((e_2, \ldots, e_n)\)
  (index spans set of powers used in database building)

Output parameter \((s_1)\)
- axle temperature
  (index spans set of points stored in database)
Singular Value Decomposition

**SVD factorization**

\[ A = U\Sigma V^T \]

- \( \Sigma \) diagonal, \( U \) and \( V \) orthogonal
- allows compression (optimal low–rank approximations)
- gives *modal* information (on rows and columns)

**Higher–Order SVD (HOSVD) factorization**

\[ A = S \times_1 U^{(1)} \times_2 U^{(2)} \cdots \times_N U^{(N)} \]

- allows compression (although \( S \) not diagonal)
- gives *modal* information (on each input variable!)
- (one variable) interpolation (using \( U^{(j)} \) columns) to predict for new input variables
Interpolation using HOSVD

Interpolation in variable related to $l$–th index

$$\mathcal{P}(x^{(l)}) = \tilde{S} \times_1 U^{(1)} \cdots \times_j \Pi(x^{(l)}) U^{(l)} \cdots \times_N U^{(N)}$$

$$P(x_1^{i_1}, \ldots, x_l, \ldots, x_N^{i_N}) =$$

$$= \sum_{j_1=1}^{N_1^T} \sum_{j_2=1}^{N_2^T} \cdots \sum_{j_N=1}^{N_N^T} s_{j_1j_2\ldots j_N} U^{(1)}_{i_1,j_1} \cdots (\sum_{k=1}^{N_l^T} \alpha_k(x_l) U^{(l)}_{k,j_l}) \cdots U^{(N)}_{i_N,j_N}$$

where:

$$\sum_{k=1}^{N_l^T} \alpha_k(x_l) U^{(l)}_{k,j_l}$$ interpolation operator on table $\{(x_l^{i_l}, U^{(l)}_{i_l,j_l})\}_{i_l=1}^{N_l^T}$
### Database building
- residence times: 540, 720 and 900 s.
- group I burners powers: 50%, 75% and 100%
- group II burners powers: 50%, 55% and 60%

### Database exploitation
- no compression used
- axle heating predicted for:
  - residence time: 630 s.
  - group I burners power: 85%
  - group II burners power: 58%
- prediction computation time $\simeq 0.1$ s
Axle heating prediction (central position)

Direct numerical simulation  DB/HOSVD prediction

Maximum error below 5 K.
Inverse design

Inverse design problem formulation
Minimization of a functional based on:
- quality of heat treatment
- energy consumption
- other parameters

Solving inverse design problems
- huge cost using direct numerical simulation
- affordable cost using a HOSVD approach
- easy computation of (exact) derivatives
- (on–going) implementation of trust region techniques using a conjugate gradient (Steihaug–Toint) algorithm
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Reheating furnace operation

Furnace operation slave to rolling train operation (programmed and non-programmed events)

usual variation of operation conditions (time span of 4 hours)
### Billets heating modelling

#### (Single) Billet heating model

\[
\rho c_p \frac{\partial T}{\partial t} - \text{div}(k \vec{\nabla} T) = 0 \quad \text{in } \Omega \times (t_{in}, t_{out})
\]

\[
T(\vec{x}, t_{in}) = T_{in} \quad \text{in } \Omega
\]

\[-k \frac{\partial T}{\partial n} = q(t) \quad \text{on } \partial \Omega \times (t_{in}, t_{out})
\]

#### Remarks

- \(q(t)\) introduces non–local (and non–linear) behaviour
  - \(q(t) = q_{conv}(t) + q_{cond}(t) + q_{rad}(t)\)
- Assembling of \(q(t)\) depends on (variable) billet position
- \(\rho c_p\) and \(k\) are temperature–dependent
### Simplified billets heating model

#### Reduction to a linear and local problem

Heating modelling over \((t_n, t_{n+1})\) with \(t_{n+1} = t_n + \Delta t\):

\[
\rho c_p \frac{\partial T_{n+1}}{\partial t} - \text{div}(k \nabla T_{n+1}) = 0 \quad \text{in } \Omega \times (t_n, t_{n+1})
\]

\[
T_{n+1}(\vec{x}, t_n) = T_n(\vec{x}, t_n) \quad \text{in } \Omega
\]

\[
-k \frac{\partial T_{n+1}}{\partial n} = q_n \quad \text{on } \partial \Omega \times (t_n, t_{n+1})
\]

with \(\rho c_p\), \(k\) and \(q_n\) computed from temperatures at \(t_n\)

#### Discretization with small \(\Delta t\)

- control of modelling error
- stability of resulting algorithm
Furnace walls heating modelling

Furnace walls heating model

\[ \rho_e c_{pe} \frac{\partial T_e}{\partial t} - \text{div}(k_e \vec{\nabla} T_e) = 0 \]

with boundary conditions

\[ -k_e \frac{\partial T_e}{\partial n} = h(T_e - T_\infty) \quad \text{on } \Gamma_{\text{ext}} \]

\[ -k_e \frac{\partial T_e}{\partial n} = q_{\text{conv}} + q_{\text{cond}} + q_{\text{rad}} \quad \text{on } \Gamma_{\text{int}} \]

and suitable initial conditions

Reduction to a linear and local problem

- same approach used for billets heating
Heat flux (to billets and furnace walls) segregation

Flux segregation
On every boundary of billets and structure:

\[ q = q_{\text{conv}} + q_{\text{cond}} + q_{\text{rad}} \]

- \( q_{\text{conv}} \): convection heat flux (from/to gases)
- \( q_{\text{cond}} \): conduction heat flux (through hearth)
- \( q_{\text{rad}} \): radiation heat flux

Modelling of segregated fluxes
- \( q_{\text{rad}} \) computed from temperatures of participating surfaces
- \( q_{\text{conv}} \) and \( q_{\text{cond}} \) correlated with actual powers (so far!)
Computing radiation flux

Energy balance on each burner

- instantaneous power (could be zero) used
- temperature assumed homogeneous over each burner

\[ \dot{W}_{\text{burner}} = \sum_{k=1}^{N_{\text{burner}}} (q_{\text{net}}^k + \rho \Delta h \vec{v} \cdot \vec{n} A_k) \]

Treatment of gaps (absence of billet charge)

- Gaps modify Gebhardt matrix (non–affordable approach)
- Instead neutral radiation surfaces (each gap) introduced

\[ 0 = \sum_{k=1}^{N_{\text{hollow}}} q_{\text{net}}^k \]
Solving heating model (billets and furnace walls)

**Galerkin method**

- Choice of a base \( \{ \Phi_i(\vec{x}) \}_{i=1}^{N_{\text{base}}} \)
- Find \( T_h(\vec{x}, t) = \sum_{i=1}^{N_{\text{base}}} \alpha_i(t)\Phi_i(\vec{x}) \) such that

\[
\sum_{i=1}^{N_{\text{base}}} \frac{d\alpha_i}{dt} \int_\Omega \rho c_p \Phi_i \Phi_j \, d\Omega + \sum_{i=1}^{N_{\text{base}}} \alpha_i \int_\Omega k \vec{\nabla} \Phi_i \cdot \vec{\nabla} \Phi_j \, d\Omega = \int_{\partial\Omega} q \Phi_j \, dS \quad \forall j = 1, 2, \ldots N_{\text{base}}
\]

**Matrix form**

\[
M \frac{d\vec{\alpha}}{dt} + K \vec{\alpha} = \vec{b}
\]
A reduced order model

Alternative I: finite element method
- \( N_{\text{base}} = \) mesh degrees of freedom
- characteristic values (number of nodes):
  - billets: 50–500 (each billet)
  - furnace walls: 10000–15000
- real time implementation unfeasible (or very challenging)

Alternative II: Galerkin–Proper Orthogonal Decomposition
- An empirical base is extracted from a set of (snapshots)
- \( N_{\text{base}} \) can be quite small:
  - 4–6 modes (furnace walls and each billet)
Implementation of a reduced–order model (ROM)

**Generation of a ROM basis**

POD computation from a set of *snapshots*
- billets: heating under *nominal* steady operation
- furnace: steady temperature under different regimes

**Assembling/integration of Galerkin–POD evolution equations**

Preprocessing of:
- mass and stiffness *modal* matrices
- projection operator to compute modal charges
- fundamental matrices of ODE’s system
**Real time implementation**

### Dynamical model (executed every minute)

Input variables (data acquired during last minute):
- actual power of each burner (sampled every second)
- time(s) of movement of walking beam system
- time(s) of charge of new billet(s)
- temperature of charged billet(s)

Initial values:
- Mean temperature of billets (beginning of the last minute)

Output variables:
- Mean temperature of billets (end of the last minute)
Preliminary validation

Reheating furnace in (hot) rolling train

- total furnace power: 80 MW
- mean power during test I: 28 MW
- mean power during test II: 37 MW

Computational details

- computation time: 0.5–0.7 s
- used memory peak: below 65 MB
Preliminary validation: test I

Dynamical model prediction vs thermocouple signals
Preliminary validation: test II

Dynamical model prediction vs thermocouple signals

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Numerical modelling under steady conditions

**Direct numerical simulation**
- accurate numerical predictions
- implementation with free software tools
- fast predictions using simulation databases

**Optimization of operation conditions**
- affordable optimization using DB/HOSVD models
Numerical modelling under dynamical conditions

Numerical simulation tool

- implementation in a real time tool
- poor numerical predictions under very variable conditions

Reduction of fuel consumption

Regulation using a previous model implementation

![Chart showing gas consumption over time](chart.png)
On-going and future work

Optimization of operation conditions (under steady condition)
- Implementation of trust region methods with DB/HOSVD models

Numerical modelling under dynamical conditions
- reduced order modelling of thermofluid dynamics
- implementation of adaptive control techniques